AN ANALYSIS OF DISTRIBUTED COMBAT SYSTEMS

Keith Jude Ho
Captain, Singapore Army
BA(Hons), MA, Cambridge University, UK , 1997

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Author: ______________________________
Keith Jude Ho

Approved by: ______________________________
Dean Wayne P. Hughes, Jr. , Thesis Advisor

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Professor Thomas Lucas, Co-Advisor

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ABSTRACT

This thesis employs Campaign Analysis techniques to examine the potential benefits of distribution of combat potential within a fleet. The term distribution refers to the allocation of a fixed amount of combat potential among fleet. To investigate the effects of distribution, the fleet size is varied and pitted against a hypothetical enemy.

In all simulations and analyses, results indicate that in most situations, a distributed fleet (one where the combat potential of the fleet is spread among a large number of ships) outperforms a concentrated fleet (one where the combat potential is spread among a fewer number of ships); where performance is measured by the number of enemy ships put out of action as well as the number of one's own ships which are put out of action by enemy missiles. While the advantages of distribution are appealing, there are two main aspects of the distributed fleet which warrant careful attention and design: communication, command and control infrastructure and logistical support.

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The author would like to dedicate this work to the memory of all the innocent people, whose lives were tragically ended on the morning of 11th Sept , 2001.

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EXECUTIVE SUMMARY

Distribution increases the **force effectiveness** of a fleet. Quantitative analyses using Lanchester Models and Naval Salvo Models indicate that a fleet with its **combat potential** concentrated in a few ships has an inferior force effectiveness when compared to a fleet in which the same amount of combat potential is spread among many smaller ships.

One of the most appealing aspects of the **distributed fleet** is that it displays a high degree of **stability** or **robustness** in combat and is able to accomplish its missions with a reasonable degree of effectiveness even after sustaining losses. In contrast, the fleet whose fighting strength is concentrated in a few warships is highly unstable and is unlikely to accomplish its mission because of a catastrophic capability reduction in the event where one or two high value ships are put out of action.

An appropriate measure for stability is to calculate the *percentage of total fleet firepower lost per* **leaker** *missile*[1]. The distributed fleet's superior performance is due to the fact that the aggregated staying power[2] of a distributed fleet is higher than that of a concentrated fleet.

Another attractive feature of the distributed fleet is its ability to mass or disperse its forces as the situation dictates. This allows the distributed fleet to operate in a variety of scenarios, giving the fleet the potential to be adaptively configured to the requirements of the mission. Possible scenarios include undertaking missions where the probability of having a few ships put out of action is high e.g., littoral warfare and missions where the use of high-value assets are an overkill e.g., drug interdiction operations and anti-piracy operations[3].

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[1]The term *missile* here is used as a generic term, it may also be thought of as a torpedo.

[2]The section on staying power is discussed in detail in Chapter IV.

[3]It is important to note that this thesis does not argue against the development and construction of large, high-value ships, rather it aims to show that there are significant benefits associated with distributed forces and that a balance between a mix of high value assets and distributed assets is

For the future battlefield, the distributed fleet, being more efficient in its use of command and control resources, is well suited for the concept of Network Centric Warfare [Ref. 3], the envisioned force multiplier of the future.

While the concept of distribution is appealing, there are two important aspects of the distributed force that warrant careful consideration and design–the command and control architecture and the logistic infrastructure. The nature of distribution implies that there will be many assets to control. Therefore, a robust command and control architecture is required to ensure the distributed assets act in a synergistic manner. Secondly, small, fast and lethal combatants form the core of the distributed force. Such combatants have limited self-sustaining capability and must be supported externally (e.g., fuel, ammunition, etc.). A robust and efficient logistic infrastructure is essential to fulfilling this requirement. It is imperative that these elements of logistic and command and control do not themselves turn out to be vulnerable **"centers of gravity"**, and thus reducing the robustness of the distributed fleet.

In addition to the quantitative analyses, historical records show that distribution of power within a fleet or force has tremendous benefits. The Yom Kippur War reaffirms this fact. Finally, many modern organizations are exploiting the advantages of distribution to increase their combat effectiveness and achieve significant results against seemingly overwhelming superior forces. Guerrilla organizations and terrorists networks are examples of such organizations.

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advantageous to the Navy.

I. INTRODUCTION

This thesis uses Campaign Analysis techniques to analyze the effects of **distribution**[1] of firepower among naval forces.

The following three chapters discuss three models, the first of which is a conceptual model based on network analysis. The other two are commonly used combat models—The Lanchester Model and the Naval Salvo Model. In Chapter V, Logistics Estimates For a Distributed Force, we discuss the size and number of logistics ships required to support a notional distributed fleet. The remaining chapters are devoted to qualitative discussions on the potential benefits of a distributed fleet and examples of modern organizations that exploit the advantages of distribution to increase their combat effectiveness.

A. BACKGROUND

The motivation for this thesis stems from VADM. Arthur Cebrowski's and CAPT. Wayne Hughes' vision of small littoral combatants, known as *Streetfighters*. The air cover required to support the *Streetfighters* was conceptualized to be provided by a fleet of small aircraft carriers. The author is part of the project team that is investigating the feasibility of this concept. This thesis forms part of the overall concept study to demonstrate the potential advantages of distribution.

1. Rumble in the Littorals

At present, there are two sides to the debate. There are those who advocate a George Foreman or a Great White Shark-styled fleet—big, extremely powerful, single punch killers; and those who advocate the Mohammed Ali or a Piranha-styled fleet—

[1] The term distribution refers to the distribution of a fixed amount of combat potential among a fleet. We will compare two types of fleets—A fleet that is made up of a few heavily armed ships, and a more distributed fleet, with more but smaller, and less heavily armed ships. See Appendix A for more details.

highly mobile, relatively powerful, combination punch killers. Two quotations are provided to capture the essence of the argument.

> *"It is difficult to see how the Crossbow[2] forces working in the littorals, with air vehicles capable of 100 to 200 miles combat radii, project greater power or influence than forces operating 100 or more miles offshore, with air vehicles capable (of) 400 to 500 miles combat radii."*

— PEO, Aircraft Carrier[3]

> *"In evaluating the designs of warships, American systems analysts almost invariably use deliverable combat potential as the decision criterion. Since a large ship enjoys economies of scale, it will carry more fuel, ordnance, aircraft, or Marines than several smaller ships of the same total cost. The analytical conclusion is therefore "Big is Better." ......If a 60,000-ton ship carries 20 times the payload of a 3,000-ton ship but can only take three or four times as many missile or torpedo hits as a small one before it is out of action, then that is a substantial disadvantage offsetting its greater payload."*

— CAPT. Wayne Hughes [Ref. 9]

These two quotes are not mutually exclusive. Both aims may be achieved, as aptly summarized in the following quote,

> *"I know that some find it hard or even distasteful to imagine a Navy with smaller ships. But it is harder and more repugnant to imagine a Navy rendered irrelevant by a focus on yesterday's missions, or shrunken to Lilliputian proportions by a tunnel-visioned fealty to large platforms.*
>
> *Capability is vital. But to take the argument to its extremes, a single immensely capable ship is not a Navy. Neither is a thousand PT boats. Today's ideal force lies somewhere between, in a form I would submit has agile platforms in greater numbers than today's very capable but musclebound structure. We cannot abandon the blue water. But the war inshore is the war for which we have to prepare. After all, American interests do not end at the 20-fathom line.*

[2]The Crossbow concept is about a fleet of about 8-10 small aircraft carriers of about 10,000 tons, supporting many small surface combatants.

[3]Verbal comments made on 17 August 2001

A rebuilt Navy should be able to operate from shoreline to shoreline. On the surface, above, and below. That will require a range of ships. Small ships, like the "Streetfighters" advocated by the President of the Naval War College, Admiral Art Cebrowski. Medium ships, like the Navy's DD-21, to provide cover for the ships inshore but able to keep station with battle groups as needed. "

—Congressman Ike Skelton [4]

B. AIM OF THESIS

The aim of this thesis is to study the potential benefits of distribution on the force effectiveness of a fleet and generate decision support data to aid decision makers in designing the composition of a fleet.

C. METHODOLOGY

This thesis uses Campaign Analysis techniques and models to evaluate the performance of a distributed fleet. A Network Model is used to investigate the efficiency of connections in a distributed fleet. Lanchester Models and Naval Salvo Models are used to investigate the force effectiveness of a distributed fleet in simulated engagements. Common parameters and measures of performance are used to compare results from all models.

1. Campaign Analysis

"Campaign Analysis is a true mix of the art of war, strategic planning, and, tactical knowledge."

— CAPT. Wayne Hughes

Campaign Analysis is the **most** difficult of all Operations Analysis. It attempts to simplify complex military operations to yield valuable insights for decision

[4]This remark was made during an evening speech on 20 September 2000 before the Business Executives for National Security. Congressman Skelton was then a Ranking Minority Member of the House Armed Services Committee.

makers and military planners. This is accomplished through the use of *abstraction*[5] and *modeling*[6]. Campaign Analysis consists of five processes, which are depicted in Figure 1.

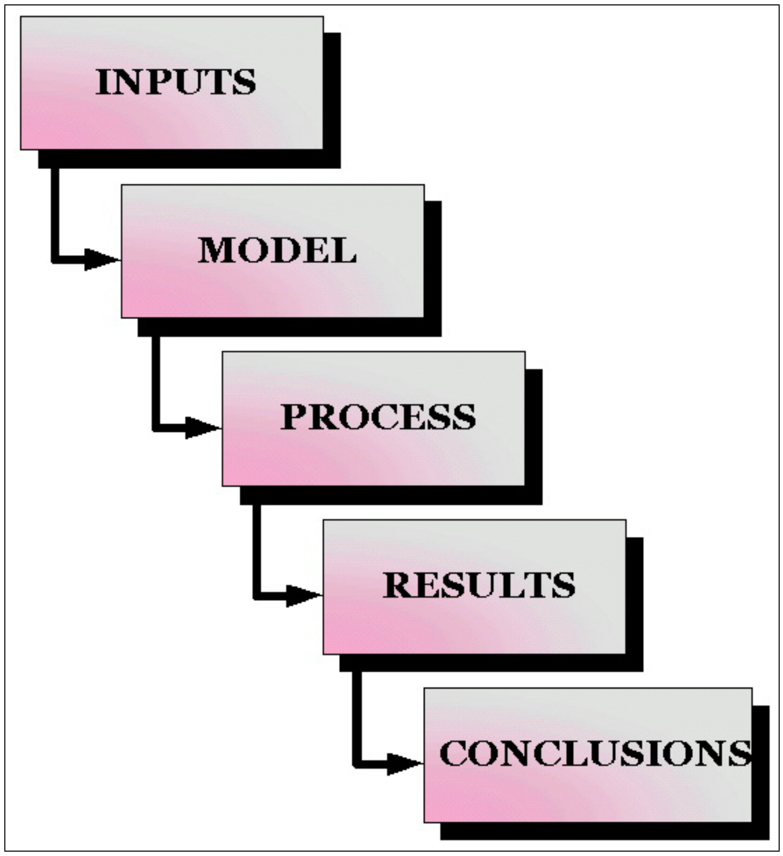

Figure 1. Five Core Processes of Campaign Analysis

From Figure 1, an analysis requires inputs or parameters, which the model will process. We will discuss the inputs and parameters of our study in the following section. Results generated by the model will indicate the performance of a particular combination of parameters, and are heavily dependent upon the type of model used. Lessons from the conclusions will serve as decision support data for military planners.

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[5]A process where analysts aggregate combat processes and express them as functions of selected parameters.

[6]The process of analytically expressing the interaction between the parameters and their effects on a synthesized outcome.

2. Process and Models of Campaign Analysis

Models and processes of Campaign Analysis may be generalized into five main categories. They are:

- **White Papers**: Use of many descriptive models, with little use of analytical equations in the explanations, but in the best works, the descriptions are based on many complex computations.
- **Mathematical Statements**: Models of this nature use mathematical equations to provide insights. Such models may be deterministic or stochastic. Models of this nature are IF-THEN statements. For example, IF 5 missiles are launched against a ship that has 3 anti-missile missiles and can withstand a maximum of 2 missile hits before being put out of action, THEN the ship will be put out of action from the salvo of 5 missiles launched at it. The Lanchester Model and the Naval Salvo Models are examples of such models. This thesis uses these two models extensively.
- **"Closed" Simulations**: Models of this nature are usually based on mathematical equations, expert rules and statements. They are usually programmed into a computer and run through a large number of iterations.
- **War Games**: This method of Campaign Analysis pits a human being against, either another human being or a computer which has expert knowledge from its previous games with other players. Such systems help military planners to validate new strategies, thoughts, and tactics. As it does not involve the use of actual force elements, it is usually used as a testing ground for new ideas.
- **Field Experiments**: Involves the use of live deployments, troops and exercises. Such methods are usually used as a final test of War Games that have proved successful. However, such methods are expensive, time-consuming and difficult to replicate.

Further discussions on the various types of Warfare Analysis are documented in *Military Modeling for Decision Making* [Ref. 11].

With the advent of modern computing power, the temptation is to develop high-resolution models. In general, the more detailed a model is, the more inputs it requires. Many of the inputs, or parameters that affect warfare are not easily obtained. Data collection for many high-resolution models may stretch for an unacceptably long period of time. Furthermore, the standard deviation of most of such data collected is usually quite substantial. If the results of a model hinges on many such parameters, the model is less robust and may exhibit instability. There is an endless list of input parameters that affect combat. A short children's rhyme illustrates the highly complex nature of combat, and dilemma of the campaign analyst.

For want of a nail, the shoe was lost,
For want of a shoe, the horse was lost,
For want of a horse, the rider was lost,
For want of a rider, a message was lost,
For want of a message, the battle was lost,
For want of a battle, the kingdom was lost,
And all for the want of a nail

The art of Operations Analysis is in selecting only the parameters of interest—among the nails, horses, riders, messages, and battles—and abstracting the complex reality into a simple function of selected parameters. The science of Operations Analysis processes these parameters to generate numerical indicators of performance. The lessons extracted from the results are dependent upon the skill, experience, and knowledge of the analyst. This thesis uses robust and well-established models—the Lanchester Model and the Naval Salvo Model—with minimal, but significant parameters to provide simple, insightful and transparent lessons. We will next discuss the parameters of interest.

3. Input Parameters

The parameters of interest that will be used as inputs to all models are:

- **Fleet Size**: Refers to the total number of ships which make up the fleet.
- **Staying Power**: The number of missiles required to put a ship out of action. This parameter is implicitly expressed in the Lanchester Equations but is explicitly expressed in the Naval Salvo Equations.
- **Offensive Capability**: The damage that a unit is capable of inflicting on an enemy. In the Lanchester Models, this is measured in the number of enemy ships that an opposing ship can put out of action per unit time, and is referred to as the *attrition coefficient*.

In the Naval Salvo Model, this is measured in the number of offensive anti-ship cruise missiles (ASCM) that a ship carries, and will be commonly referred to as *offensive firepower*. In both cases, the linear sum of each individual ship's capability will represent the fleet's total offensive capability. A more detailed discussion on these parameters will be found in the respective chapters where they are used.

- **Defensive Capability**: The number of incoming missiles that a ship is able to destroy/defend against. Measured by the number of anti-missile missiles (e.g. Rolling Airframe Missiles, Barak Missiles) that a ship possesses, and will be commonly referred to as *defensive firepower*. This parameter is implicitly embedded in the Lanchester model but explicitly expressed in the Naval Salvo Model. In both cases, the fleet's total defensive capability is a linear summation of each individual ship's defensive capability. These parameters will be discussed in detail in the respective chapters that they are used.

To make a fair comparison between distributed forces and concentrated forces, we have imposed a condition on all our analyses: The fleet's **TOTAL** capability—defensive and offensive—is kept constant. Our aim is to show that with the same amount of defensive and offensive hardware, it is better to distribute the hardware to more combatants. This artificially imposed condition gives rise to a few implicit assumptions:

- A fleet designed with big ships will have fewer ships than a fleet designed with smaller ships. This arises because fixed amount of firepower or combat potential can be carried by a fewer numbers of large ships.
- A larger ship has greater offensive and defensive firepower as compared to a smaller ship.

Staying power is an important parameter that affects all our calculations. In general, we will assume that a larger ship will have a higher staying power than a smaller ship. In Chapter IV, The Naval Salvo Model, there is a detailed discussion on this parameter. There are other minor parameters that will be explained as they are used. This thesis investigates the force effectiveness of the fleet when these parameters are varied.

4. Measure of Performance

Combat models provide numerical solutions as an indication of force effectiveness—how well a force performs. But how do we quantify the force effectiveness of a fleet? We quote CAPT. Hughes again,

> *"...success was measured in ship casualties and a comparison of the numbers put out of action on the two sides... "* [Ref. 9]
>
> — CAPT. Wayne Hughes

The prime measure of performance that this thesis uses is:

- **Ships Put Out of Action**: This refers to the **number** of ships which have been disabled by a firepower kill, to the extent that they can no longer contribute to the mission. Variants include fractional fleet losses.

There are other variants of measures of performance used in this thesis and they will be discussed in the respective chapters where they are employed.

D. ORGANIZATION

The first part of this thesis concentrates on the quantitative and simulation aspects of the study. The latter chapters are qualitative discussions. *Chapter Two* presents a network-based model analysis. It uses established theories of network analysis to compare a distributed fleet with a concentrated fleet in terms of *control nodes* and complexity.

Chapter Three makes the comparison using classic Lanchester Equations. This chapter's main focus is on the relationship between winning percentage and the number of combatants used. It keeps the total offensive capability constant and varies the numbers of combatants used. A detailed discussion and explanation of Lanchester Equations and its history may be found in *Lanchester Models of Warfare* [Ref. 20].

Chapter Four uses the Naval Salvo Equations as a basis for analysis. The mechanics of this equation may be found in Chapter 11 of *Fleet Tactics and Coastal*

Combat [Ref. 9]. A simple deterministic model is first discussed. It is then developed into a simple stochastic model, and the "instability" of fleet is discussed. The same chapter introduces a new measure of performance to evaluate various configurations, **Percentage of Total Fleet Firepower Lost Per Leaker**. The simple stochastic model is further developed into a slightly more complex stochastic model. This chapter concludes by presenting the results of a simple campaign analysis exercise conducted by students of the Naval Postgraduate School.

Chapter Five examines the logistic requirements to support a distributed fleet. Using an easily understood model, an estimate of the number and size of the logistic ships required to support the distributed fleet is obtained. The distributed fleet that is used in this chapter is the notional distributed fleet that was used in the same campaign analysis exercise mentioned above.

Chapter Six discusses some aspects of a distributed force in a qualitative manner. It includes discussions on real world examples of distributed forces and their effectiveness. Finally, the last chapter summarizes all the conclusions from preceding chapters.

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II. THE NETWORK MODEL

"Go separately, hit in unison!"

— Unknown

A. INTRODUCTION

This chapter presents a model of a distributed force from a network view. Using this model, this chapter will show that a distributed force and a distributed command and control structure is more efficient and reliable than a single concentrated force with a centralized command and control structure.

1. Organization

The first section is a glossary of terms used in the model and the assumptions of the model. The second section is an explanation of the model, and the third section is the mathematical proof that a networked and distributed force is more efficient, and more reliable as compared, to a single concentrated force.

2. Network Model Terminology

- **SM-2**: A particular kind of missile used by the U.S. Navy[1], used against incoming missiles as well as aircraft. In the Network Model, a ship is described as having N number of SM-2s. This is not the total number of missiles that the ship has. Rather, N represents the MAXIMUM number of missiles that can be fired in a SINGLE salvo. Therefore, each position on the Network Model that has a SM-2 represented on it, represents a missile tube on board a ship. After the first SM-2 is launched, another SM-2 will be reloaded into the tube to take its place.

- **Control Nodes**: An interface between a missile and the fire control radar that directs it to a particular target. A fire control radar collects information about the target (e.g., position, velocity, IFF data, etc.), transfers the data to a particular missile, and guides it to destroy the target. The interface through which this data is sent, is termed a control node.

[1]The SM-2 is a used as a representation of a missile, it may be any other missile

- **Airborne Communications Node**: A collection of control nodes which are physically separate from the platform. It provides the linkages between platforms. The nodes need not be, but are depicted as airborne to simplify the concept.

3. Model Assumptions

The assumptions of the Network Model are as follows:

- Every missile tube on board a ship is connected such that it can be directed by any fire control radar on board a ship.
- When a fire control radar is directing a missile to its target, a dedicated connection is made from that radar to the missile. The fire control radar cannot direct any other missiles when it is already directing a missile. Only after the first missile has destroyed its target or has itself been destroyed, can the radar be re-used to direct the next missile to a target[2].

B. THE NETWORK MODEL

Figure 2 depicts a simplified view of a missile defense system on board a ship. The system is composed of two fire control radars, and two anti-missile (SM-2) missile tubes.

Each fire control radar must be able to direct either one of the missiles towards an incoming threat. Hence, missile 1 must be connected to both fire control radars, similarly for missile 2. The connections are termed *control nodes.* Figure 3 is a graphical representation of the system of connections. For example, when fire control radar 1 tracks an incoming target, and missile 1 is assigned to destroy that target, control node 2 connects fire control radar 1 to missile 1, and is now in use. No other missiles may be directed by radar 1 while missile 1 is still enroute to engaging its target.

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[2]Fire and forget or terminally guided missiles may not require the radar to guide the missile all the way to the target, but it would still require that the missile be guided at the terminal phase. The connection diagram still holds for terminally guided missiles.

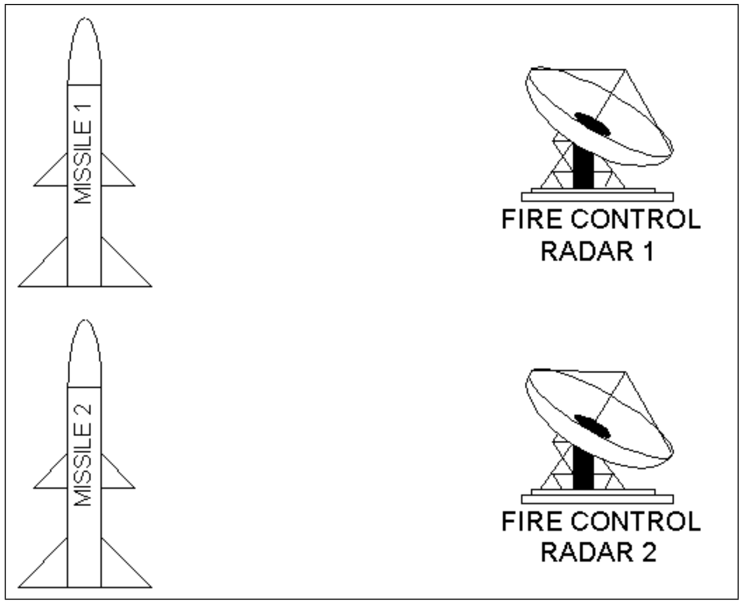

Figure 2. Simple Missile Defense System

The connections may be visualized in Figure 4. When missile 1 is directed by radar 1, control node 2 (highlighted) will be used and the dedicated connection path is established along the darkened line.

From Figure 5, it can be seen that a simple system consisting of N directors or fire control radars, with N numbers of SM-2s would require N^2 control nodes. In network theory, such a setup of connections is known as a *Crossbar Switching System.* This number of control nodes is required to ensure that every fire control radar is able to interface with each and every SM-2 missile when required. In general,

$$X_{single} = N^2 \tag{II.1}$$

Where,

- X_{single} = Total number of control nodes in a single platform;
- N = Total number of missiles tubes in a single platform;
- N = Total number radars in a single platform.

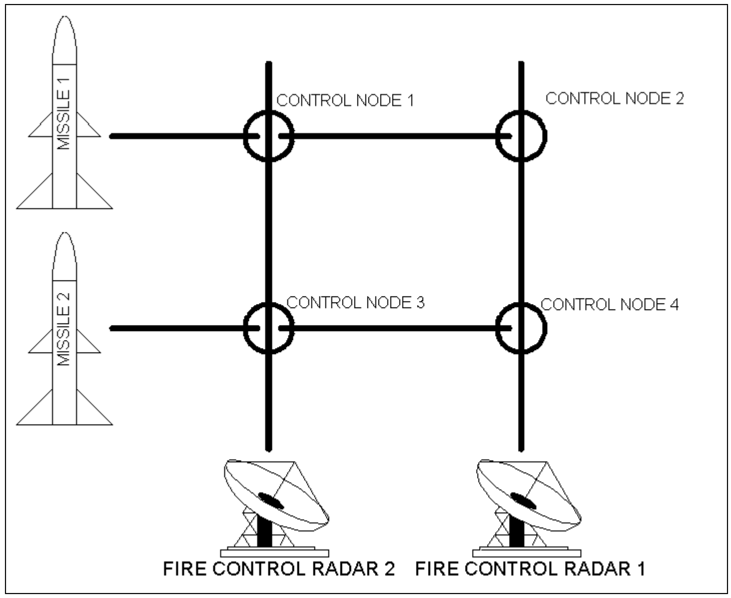

Figure 3. Connections on the Simple Missile Defense System

C. LIMITATIONS OF A SINGLE PLATFORM

The limitations of such an arrangement are as follows:

- The number of control nodes grows with the square of the number of SM-2s, increasing the complexity of the command and control system.
- The single system is more vulnerable. The loss of a single control node prevents connection between the SM-2 and the radar whose bars intersect at that control node.
- The control nodes are not efficiently used, in that, at any point of time, a maximum of N, out of the N^2 control nodes are being used.

D. THE DISTRIBUTED FIGHTING FORCE

The first part of this section will discuss the development of the model for the distributed force. The second part of this chapter will discuss the distributed fighting force from two perspectives. In the first perspective, the total number of SM-2s will be kept the same, and we will look at how a distributed fighting force can reduce the number of control nodes required. Along the same train of thought, we will investigate the number of SM-2s (and conversely, the number of targets that can

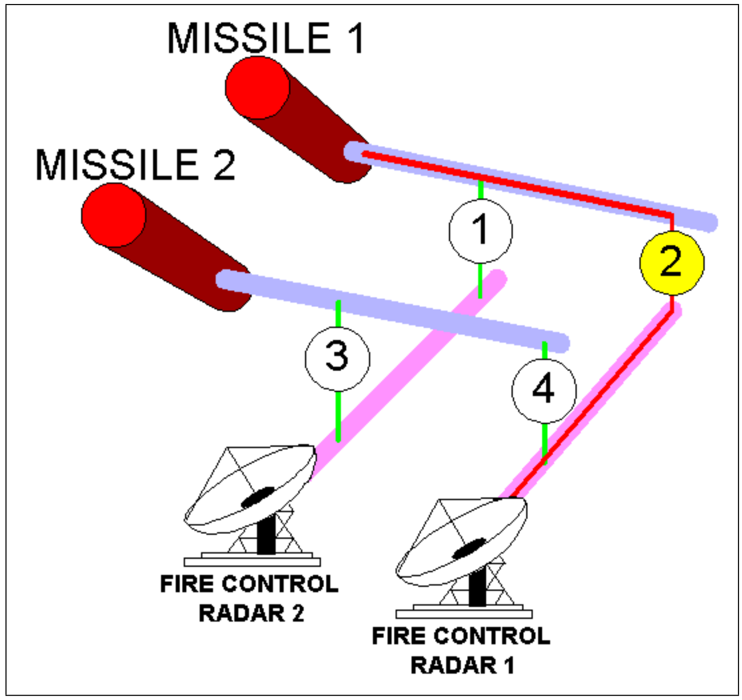

Figure 4. 3-D View of Control Nodes

be tracked and destroyed) that a distributed fighting force, with the same number of control nodes as a single entity, can control.

1. Distributed Fighting Force—Model Basics

A distributed fighting force is postulated to be one where the large, single platform fighting force is divided up into smaller units, linked by a single or multiple command and control airborne communication nodes. The feasibility and architecture of an Airborne Communication Node System are documented in a study performed by Richard Foo, *Requirement Analysis of An Airborne Communications Node (ACN) in Support of Crossbow Operations* [Ref. 7]. The configuration of the system is illustrated in Figure 6.

The basis of comparison is with a single platform and the same notation is used. Let the smaller fighting force have a total of n SM-2 missiles and n radars on each platform. The total number of SM-2 missiles for the entire fleet is still N. This implies that there are $\frac{N}{n}$ smaller platforms. A single ship of the distributed fleet is

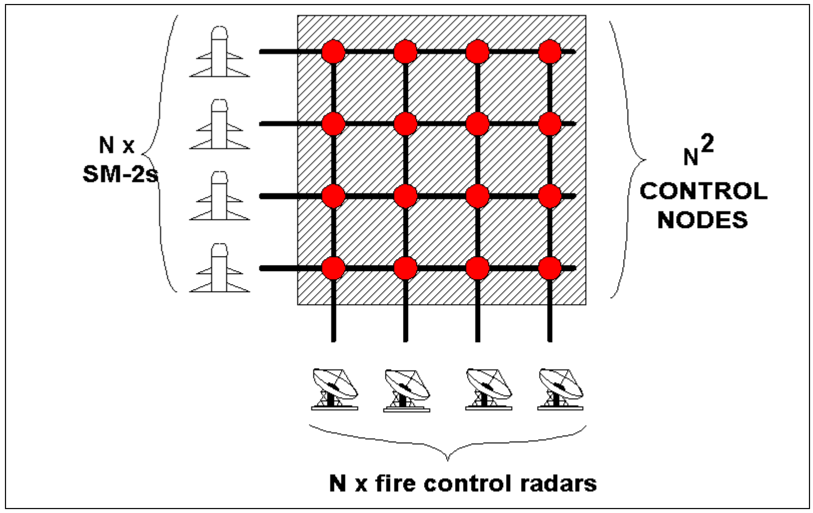

Figure 5. N Missiles and N Radars Require N^2 Control Nodes

denoted as the outlined area labeled A in Figure 6. Within the area A, the boxes on the right denote the number of radars on board a single ship and the boxes on the left denote the missile launchers on that same ship.

An airborne command and control structure links all the smaller fighting forces together, this is denoted by area B in the same figure. A system employing this structure is one where the assets aboard any one of the smaller forces can be effectively used by any of other forces in the fleet. (This is in concert with the concept of Network Centric Warfare [Ref. 3]).

The connection configuration in each small platform and each airborne communications node is a Crossbar Switching System. Each airborne node has a connection to each of the small platforms. That is, there are k (the number of airborne control nodes) lines coming out from each of small platforms (on the left hand side). Into each airborne communication node, there are $\frac{N}{n}$ inputs. Similarly, there are $\frac{N}{n}$ output lines from each of the airborne communications node (one for each platform on the right hand side).

The next step is to calculate the number of modules of airborne nodes that are required to support this structure. The basis for this calculation is a phenomenon

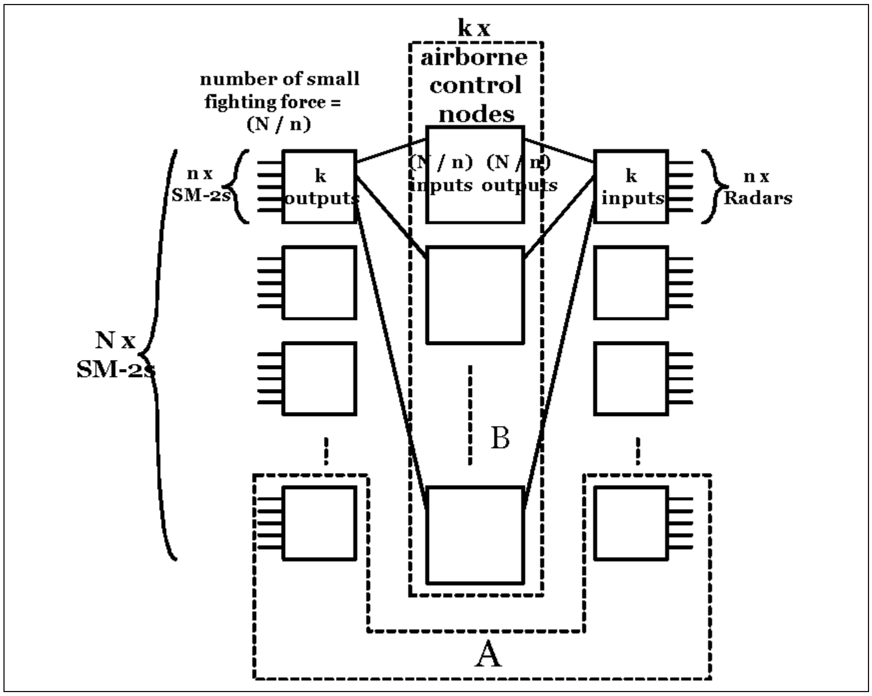

Figure 6. The Distributed Fighting Force

called “**blocking**” in network terminology. “**Blocking**” is defined as the inability to direct a SM-2 missile with a radar, a simple example will exhibit this phenomenon.

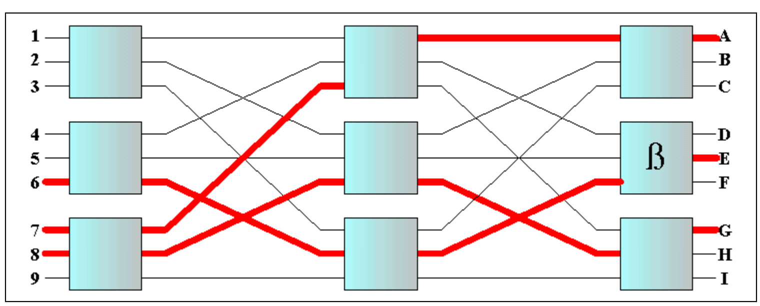

Figure 7. Example of Blocking in a Network

Looking at Figure 7, where there are 3 small fighting platforms and 3 airborne communication nodes. A situation arises when the β platform is already tracking an incoming missile and is directing SM-2 number 6 to destroy that incoming missile. The heavy lines indicate lines that are already in use. If platform β detects another incoming missile, and the only available SM-2 is SM-2 number 9, the system is then unable to respond to the threat. The implicit assumption is that only β is capable of responding to the threat. Therefore, it is clear that the number of airborne communications nodes must be increased to avoid this situation. The calculation of the number of airborne nodes is as follows:

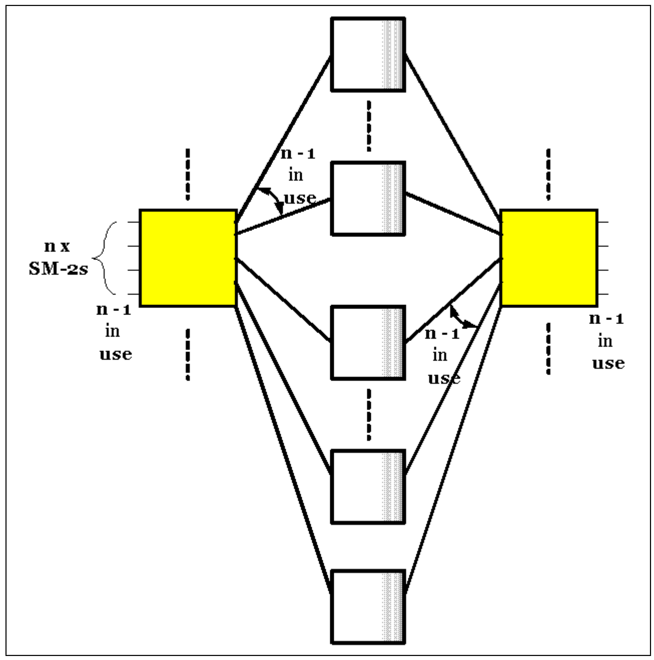

Figure 8. Number of Airborne Nodes

Referring to Figure 8, assume that all $n-1$ missiles on the platform of interest are being used and so there will be $n-1$ output lines from this platform which are in use. (Note that the radar and the missiles need not be on the same platform). Now, assume that the platform which has just detected the incoming missile and wants to direct a SM-2 to intercept this missile also has $n-1$ radars already in use (worst case analysis). And these $n-1$ incoming missiles are being matched with missiles from other platforms. Therefore, $n-1$ of the incoming lines to this platform are already in use. Therefore, we will require one additional airborne node to rectify this problem. Therefore a total of $(n-1)+(n-1)+1$ airborne nodes are required. Therefore

$$
\begin{aligned}
k &= (n-1) + (n-1) + 1 \\
\therefore k &= 2n - 1
\end{aligned}
\tag{II.2}
$$

2. Effect of Distribution on the Total Number of Control Nodes

The stage is now set to compare the number of control nodes of a distributed fighting force to a single fighting force. For a system which has N radars and N SM-2s, a single fighting force would require N^2 control nodes. With the distributed fighting force, the total number of control nodes, X_{dis}, is calculated as follows:

Referring to Figure 6, looking at a single box on the leftmost column, there are n inputs and there are k or $2n-1$ outputs from that box. Therefore the number of control nodes in that box is $n \times k$. But there are $\frac{N}{n}$ of these boxes in the leftmost column of boxes. There are the same number of control nodes in the rightmost column of boxes. Therefore the two end columns of boxes will have a total of $2 \times n \times k \times \frac{N}{n}$ control nodes. Now for the center column or the airborne modules, each center box contains $\frac{N}{n}$ inputs and $\frac{N}{n}$ outputs. Therefore each center box has $\frac{N}{n} \times \frac{N}{n}$ control nodes, but there are a total of k of these boxes in the center column. Therefore the total number of control nodes associated with the entire distributed system is:

$$
\begin{aligned}
X_{dis} &= 2 \times n \times k \times \frac{N}{n} + k \times (\frac{N}{n})^2 \\
\therefore X_{dis} &= 2Nk + k(\frac{N}{n})^2 \\
\therefore X_{dis} &= 2N(2n-1) + (2n-1)(\frac{N}{n})^2
\end{aligned}
\tag{II.3}
$$

Where,

- X_{dis} = Total number of control nodes in a distributed system
- n = Total number of missiles on board a single ship
- k = Total number of airborne modules

- N = Total number of missiles in the fleet

Some numerical examples are highlighted in Table I to see the effect of this.

Total Number SM-2s, (N)	Number of small platforms, (n)	Number of control nodes in single platform	Number of control nodes in distributed platform, (X_{dis})	Number of airborne nodes, (k)	Percentage Advantage (%)
4	2	16	36	3	2.25
9	3	81	135	5	1.67
16	4	256	336	7	1.31
25	5	625	675	9	1.08
36	6	1296	1188	11	0.92
49	7	2401	1911	13	0.80
64	8	4096	2880	15	0.70
81	9	6561	4131	17	0.63
100	10	10000	5700	19	0.57
121	11	14641	7623	21	0.52
144	12	20736	9936	23	0.48
169	13	28561	12675	25	0.44
196	14	38416	15876	27	0.41

Table I. Number of Control Nodes Comparison

From Table I, the trend shows that as more and more missiles are required, a distributed network brings increasing returns by reducing the number of control nodes required.

3. Keeping the Number of Control Nodes Constant

This section will attempt to mathematically show the increased potential of a distributed force, if the number of control nodes is the same as its single entity counter part. By keeping the number of control nodes the same, we will then calculate what is the potential number of targets and missiles that we can control using the same number of Control Nodes.

Assume that a hypothetical platform has 49 controllable assets that it can control and direct to address incoming threats. From Table I, it can be seen that a distributed system of 7 small vessels, each armed with 7 assets will only use 80% the number of control nodes as compared to a single platform, (2401 vs. 1991).

For simplicity of round numbers, let us assume that a single platform has 50 controllable assets. Using this number we find that a single platform would require 2500 control nodes. If we were to distribute these assets into smaller platforms, what structure and capability would result? Equation II.3, derived earlier for the total number of control nodes in a distributed system is used.

$$X_{dis} = 2N(2n-1) + (2n-1)(\frac{N}{n})^2$$

Substitute X_{dis} with 2500 and solve the relationship between N and n. This graphical solution is depicted as Figure 9.

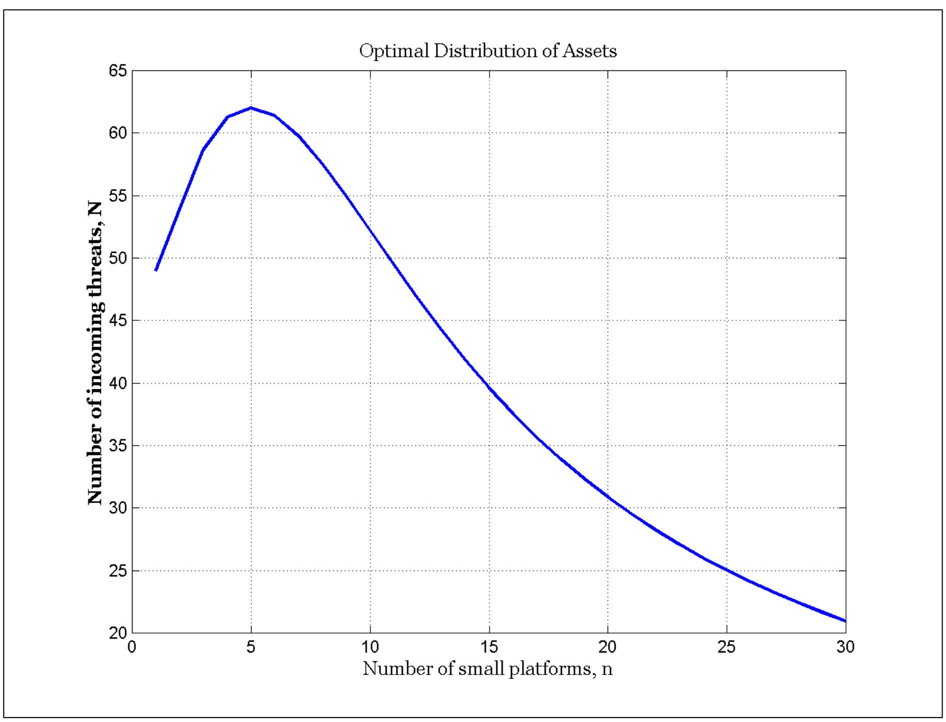

Figure 9. Distribution of Distributed Forces

Figure 9 shows that with 2500 control nodes in a system, 5 distributed platforms (each with $\frac{50}{5}$ or 10 assets) would be able to effectively direct 60 assets against 60 incoming threats (if the number of assets were actually increased from 10 to 12 per small platform, it would be able to do that, hence the use of the word *potential*). This represents a 20% (from 50 to 60) increase in controlling and tracking potential, if a distributed system is used. As an aside, a single platform trying to control 60 assets would require $60^2 = 3600$ control nodes.

Hence it can be concluded that a distributed system can achieve a more efficient use of the command and control network, or conversely, a distributed system, with the same number of control nodes as a single platform, can offer an increase of 20% in threat handling capability[3].

4. The Reliability of a Distributed Fighting Force

A look at the single platform approach shows that if one control node were disabled or destroyed, the capability of the platform is effectively reduced, as there is a void between one of the fire control radars and one of the missile pods. However by inspection of the distributed system, the connection of a specific missile to a specific fire control radar is not dependent upon a single control node, there is redundancy in the distributed fighting force, increasing the reliability and robustness of the distributed network.

5. Flexibility of Distributed Forces

The network model also highlights one important concept, i.e., the ability of a platform to deliver fire using targeting information from other platforms. In Chapter 11 of *Fleet Tactics and Coastal Combat* [Ref. 9], CAPT. Wayne Hughes develops a range-dependent model of modern naval combat. One of the postulated scenarios requires a fleet to be able to send in a part of its force, with its radars turned off (to avoid alerting the enemy—and depending on the remainder of fleet, which are placed beyond the range of the enemy's radar, for its scouting information) to be able to launch an effective first strike against the enemy.

[3]This increased capability is only realized when the number of assets are increased.

E. CHAPTER SUMMARY

The analysis shows the following points:

- Increased number of threats will require increased countermeasures. The number of control nodes and interfaces will not increase linearly, but quadratically with the number of threats and countermeasures. As the boundary between naval operations and land operations becomes blurred, the number of targets (or threats) will increase due to the addition of land-based threats (missiles, airplanes, rockets and long range artillery).

- The trend towards designing a multi-mission countermeasure (e.g., a missile that can handle air, surface and land targets) will simplify the logistics of the operation, but it would give rise to increased interfaces and control nodes, which in turn could lead to latency and delay in countermeasure response.

- A distributed system provides the opportunity for redundancy in terms of information links.

- A distributed system must be supported by a reliable and robust information infrastructure. In this analysis, we have visualized it as an airborne module. At the point of writing, a study [Ref. 7] is being conducted on the viability and architecture of a distributed airborne communication node structure. The results of which can be usefully applied to the conceptualization of this airborne command and control module.

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III. A LANCHESTER EQUATIONS PERSPECTIVE

A. INTRODUCTION

The Lanchester Equations are used here to analyze the effects of increased numbers versus increased unit capability[1] on the overall force effectiveness.

1. Organization

The first section will present the basic Lanchester Analysis. We will then develop a simple stochastic model based on the Lanchester Model and conduct a simple experiment to study the effects of distribution. The last section will look at the applicability of the Lanchester Model for modern naval combat and will also discuss how defensive powers and staying powers are implicitly built into the Lanchester Model.

2. Lanchester Equations Assumptions

Before we use the Lanchester Equations, there are a few assumptions of the model that must be understood.

- Each unit on either side is within weapon range of all units on the other side.
- Each unit is sufficiently well aware of the location and condition of all enemy units, so that it engages only live enemy units (one at a time) and attrites them at a constant rate, which does not depend on the enemy force level. When a unit enemy target is killed, search begins for a new target, with the rate of acquiring a new enemy target being independent of the enemy's force level.

These assumptions are highly debatable for a modern naval battle. Lanchester Equations find more applicability to a gun battle, as opposed to a missile exchange. Modern naval battles now resemble missile exchanges, hence results from modern

[1] In a typical Lanchester Model, the capability of a single unit of the force is captured in its *attrition coefficient*.

naval combat models using Lanchester Equations must be used with caution and a thorough understanding of the associated implicit assumptions.

3. Lanchester Analysis Terminology

The terms used are defined as follows:

- **Force Level, x and y**: The force level, measured in the number of units of the respective forces, X and Y, at a particular moment in time. Subscripts on these values denote the value of the force level at a particular point in time. x_0 denotes the initial force level at time, t=0. x_t denotes the force level at time, t, and x_f denotes the number of units left of X after the battle is terminated at time, t_f;
- **Attrition Coefficient, a and b**: The attrition coefficient of a particular force. If Y's attrition coefficient is a, it means that each unit of Y, is capable of attriting a units of X per unit time. Similarly, if X's attrition coefficient is b, it means that each unit of X, is capable of attriting b units of Y per unit time.

B. BASIC LANCHESTER ANALYSIS

Using the basic Lanchester Equations for modern combat derived in *Lanchester Models of Warfare* by James G. Taylor [Ref. 20]:

$$\frac{dx}{dt} = \begin{cases} -ay, & \text{for} \quad x > 0; \\ 0, & \text{for} \quad x = 0. \end{cases} \tag{III.1a}$$

$$\frac{dy}{dt} = \begin{cases} -bx, & \text{for} \quad y > 0; \\ 0 & \text{for} \quad y = 0. \end{cases} \tag{III.1b}$$

Where,

- x = number of units in force X;
- y = number of units in force Y;
- a = unit attrition coefficient of Y;
- b = unit attrition coefficient of X;
- t = time

The solutions to the equations are quoted from the same source [Ref. 20].

$$x(t) = \frac{1}{2}\left((x_0 - \sqrt{\frac{a}{b}}y_0)e^{(\sqrt{ab})t} + (x_0 + \sqrt{\frac{a}{b}}y_0)e^{-(\sqrt{ab})t}\right) \tag{III.2a}$$

$$y(t) = \frac{1}{2}\left((y_0 - \sqrt{\frac{b}{a}}x_0)e^{(\sqrt{ab})t} + (y_0 + \sqrt{\frac{b}{a}}x_0)e^{-(\sqrt{ab})t}\right) \tag{III.2b}$$

Using these equations, we assign a fixed sized enemy (Red, y) with a fixed attrition coefficient (a), and vary the size force (Blue, x) and attrition coefficient of our own forces (b). Red is given a force of 20 ($y = 20$) ships. Each Red unit has an attrition coefficient of 1 ($a = 1$) ship per unit time, and a standard deviation of 1 ship[2]. Blue is given a fleet of 1 ($x = 1$) to 10 ($x = 10$) ships, but at any time, the force's total attrition coefficient is constant. Blue force's total attrition coefficient is 50 of Red's ships put out of action per unit time. As an example, if Blue has 1 ship, that single Blue ship can put 50 Red ships out of action per unit time. Similarly, if Blue has 2 ships, each ship can put 25 Red ships out of action per unit time. The individual attrition coefficient of each side is modeled as a random variable with a normal distribution, to represent the uncertainty of the capability of a force. For example, if each unit of Blue (x) has an attrition coefficient of 6.25 ships per unit time (b=6.25), the actual attrition coefficient is a random variable, normally distributed, with a mean of 6.25 ships and a standard deviation of 1 ship.

Table II summarizes the forces, and individual attrition coefficients involved.

To ensure a fair comparison, we assume that the product of the Blue's attrition coefficient (a) and the number of Blue platforms (y), is a constant. For each distribution, the encounter is run 100,000 times using MATLAB. The winner of an engagement is the side that has forces surviving when the other has reached 0. The results for the 100,000 simulations are expressed as a percentage of 100,000 and the

[2] If the random variable is negative, the attrition coefficient is set to 0.

Item	Red Force	Blue Force
Force Size	20	varied from 1 to 10
Capability (1 Blue)	a∼N(1,1)	b∼N(50,1)
Capability (2 Blue)	a∼N(1,1)	b∼N(25,1)
Capability (3 Blue)	a∼N(1,1)	b∼N(16.7,1)
Capability (4 Blue)	a∼N(1,1)	b∼N(12.5,1)
Capability (5 Blue)	a∼N(1,1)	b∼N(10,1)
Capability (6 Blue)	a∼N(1,1)	b∼N(8.3,1)
Capability (7 Blue)	a∼N(1,1)	b∼N(7.1,1)
Capability (8 Blue)	a∼N(1,1)	b∼N(6.25,1)
Capability (9 Blue)	a∼N(1,1)	b∼N(5.6,1)
Capability (10 Blue)	a∼N(1,1)	b∼N(5,1)

Table II. Lanchester Parameters for Blue and Red Forces

process is repeated for each configuration. The results are summarized in the graph on Figure 10.

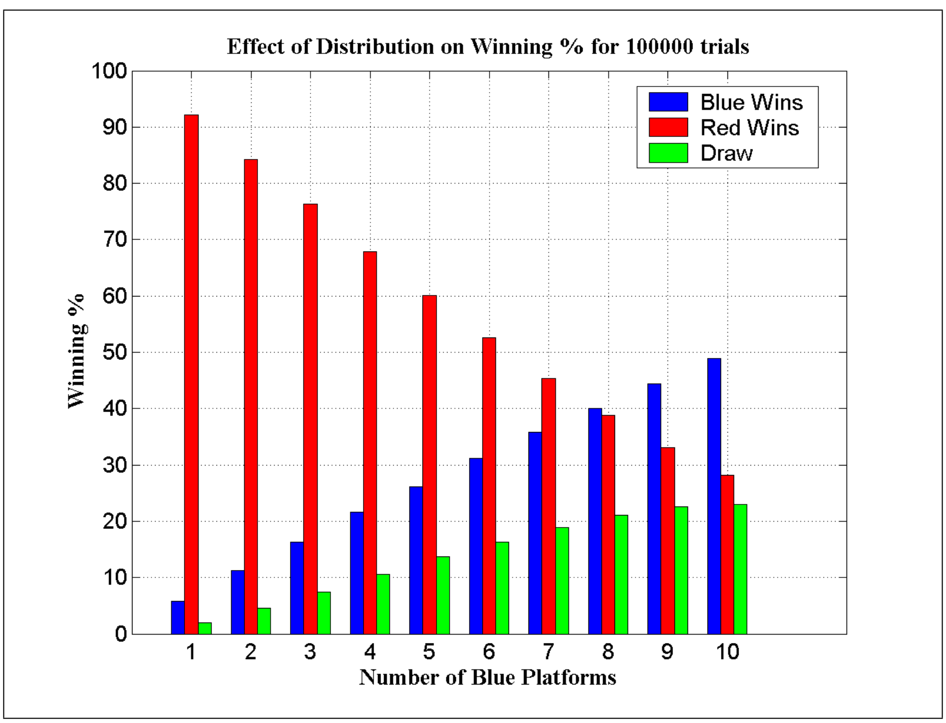


Figure 10. Effects of Distribution on Winning Percentage

1. Results and Discussions from Basic Lanchester Analysis

The general trend of the results indicates that a highly concentrated force, composed of units with high attrition coefficients, tends to perform poorly against our enemy in comparison to the distributed force. There appears to be parity when 8 units of Blue are employed against Red.

This is due to the Lanchester square law. The square law state equation for force parity is,

$$b(x_0^2 - x_f^2) = a(y_0^2 - y_f^2) \tag{III.3}$$

Assume that both sides are destroyed at the end,

$$\therefore x_f = y_f = 0$$

Therefore for force parity,

$$b(x_0^2) = a(y_0^2) \tag{III.4}$$

From Figure 10, force parity occurs when there are 8 Blue ships. For 8 ships, the attrition coefficient of each ship is $\frac{50}{8} = 6.25$. Checking Lanchester's condition for force parity,

$$(6.25)(x_0^2) = 1(20^2)$$
$$\therefore x_0 = \sqrt{\frac{400}{6.25}}$$
$$\Rightarrow x_0 = 8$$

From our results, we note that with fixed resources (constant total attrition coefficient), increased force effectiveness may be achieved by merely redistributing firepower among more units.

This phenomenon occurs because of the square law solutions that the Lanchester Equations generate. Referring to Equation III.4, the equations can be easily manipulated to show that for a fixed enemy, a proportional increase in the number of units employed is equivalent to twice the proportional increase in the attrition coefficient. Assume that the enemy force size and attrition coefficient is constant and represented by k, we calculate the percentage increase in force effectiveness when we increase the attrition coefficient (b), using the following relationship,

$$\begin{aligned} k &= bx^2 \\ \frac{dk}{db} &= x^2 \end{aligned}$$

Using the following approximation to find the change in the value of k when b is varied,

$$\begin{aligned} \frac{\Delta k}{\Delta b} &\approx \frac{dk}{db} \\ \Rightarrow \Delta k &\approx \frac{dk}{db} \times \Delta b \end{aligned}$$

But from above,

$$\begin{aligned} \frac{dk}{db} &= x^2 \\ \therefore \Delta k &\approx x^2 \times \Delta b \end{aligned}$$

Dividing the equation by k, and using the expression that $k = bx^2$,

$$\begin{aligned} \frac{\Delta k}{k} &\approx \frac{x^2 \times \Delta b}{k} \\ \Rightarrow \frac{\Delta k}{k} &\approx \frac{x^2 \times \Delta b}{bx^2} \\ &= \frac{\Delta b}{b} \end{aligned}$$

Similarly, an increase in the force size (x), with attrition coefficient (b) kept constant,

$$\begin{aligned} k &= bx^2 \\ \frac{dk}{dx} &= 2xb \\ \Delta k &\approx \frac{dk}{dx} \times \Delta x \\ \Rightarrow \frac{\Delta k}{k} &\approx 2\frac{\Delta x}{x} \end{aligned}$$

A proportional increase in the force effectiveness $\frac{\Delta k}{k}$ can be achieved by an increase in attrition coefficient by a certain percentage OR by an increase in the force numbers by half that percentage. This heavy dependence of numbers over force effectiveness is not an academic peculiarity; it is a real phenomenon in combat. There are two logical reasons why this is so: [Ref. 22]

- The extra unit fires at the enemy,
- The extra unit dilutes the enemy's fire against the units already in battle.

An increase in the attrition coefficient does not dilute the enemy's fire on our own forces, but an increase in force numbers does. In diluting the enemy's fire, we are effectively lowering its attrition coefficient and increasing our own.

Attrition coefficients are not functions of weapons capability and missile accuracy alone. They are affected by the nature of the terrain and environment in which the battle is being conducted. For example, in land combat, attrition coefficients are determined by the posture of the forces in conflict. A defending side is usually assumed to have a 3:1 advantage over the attacking side. The reasons are that the defending side is usually more familiar with the terrain and the defending side has the advantage of having pre-positioned forces in the battle ground.

In the battle of Iwo Jima, the Lanchester Equations are strikingly accurate. We digress to the historical example where a side with the inferior attrition coefficient overcame the enemy through use of numbers. The U.S Marines had about 70,000 troops launched against approximately 28,000 Japanese troops who were dug in. *Engel* postulated that with a strongly prepared defense, the Japanese had a 5:1 [Ref. 5] attrition coefficient advantage. Substituting these values into Equation III.3, we calculate that the U.S. casualties will be about 31,000 troops. (Assume that all the Japanese were killed). The actual outcome was that a total of 28,000 Marines were wounded or killed.

In a littoral context, the coastal navy is in a defensive position. It has better knowledge of the coastal waters, and the option of pre-positioning forces in the vicinity (e.g., mines). Therefore, it will most likely have the advantage in attrition coefficient over most attacking fleets. In such a case, an attacking fleet can make up for its inferior attrition coefficient by distributing its combat potential among more ships.

2. Summary of Lanchester Analysis

Lanchester Analysis shows that we can increase force effectiveness, without actually increasing the total firepower of a fleet, but by merely redistributing the combat potential among more platforms. However, Lanchester Models are used more often in land warfare, where the conditions of continuous fire are more applicable. Although modern naval warfare is mainly characterized by missile warfare, we note the two following developments:

- Long range missiles are big and so are of a finite number on board a ship. When expended, a ship has to rely on its aircraft and guns for firepower and protection. In the case of aircraft carrying "smart bombs", and guns, the battle then will bear more resemblance to Lanchester Equations.
- As technology progresses, countermeasures against missile systems will increase. Technology has, however, not been able to produce a suitable defense against a "blind and dumb" 16-inch shell, or a "blind and equally dumb" 200 knot supercavitating torpedo. When such "line of sight" weapons do return to the battlefield, naval warfare may, once again, approach its Lanchester roots.

C. USE OF LANCHESTER ANALYSIS FOR NAVAL COMBAT

The Lanchester Model of combat is an offensive-biased model and does not explicitly take into account defensive capability or survivability. One can assume that the attrition coefficient may have been derived in the following manner: A rifle or naval gun fires 2 shots per minute. It takes 2 shots[3] to put a soldier or ship out of action. Consequently, the attrition coefficient would be 1 soldier or ship per unit time—OR—the same attrition coefficient could be the result of 4 shots fired per minute, but every other shot is defeated by some abstract defense mechanism. The defensive capability and the staying power are thus embedded into the Lanchester attrition coefficients but are not explicitly expressed in the main equation.

Though F.W. Lanchester took credit for his modeling equations in 1914 when he published his hypotheses on attrition in warfare, there were similar approaches already by *J.V. Chase* [Ref. 4], and the Russian, *Osipov* [Ref. 18]. Chase was in the U.S. Navy and his papers were classified till 1972. It is significant that Chase's analysis had in fact taken ship *survivability* into account. A modified version of Chase's analysis is documented in Appendix C of *Value of Warship Attributes in Missile Combat* [Ref. 12]. Chase's equations are,

$$\frac{dX(t)}{dt} = \frac{ay(t)}{x_1} \qquad \text{(III.5a)}$$

$$\frac{dY(t)}{dt} = \frac{bx(t)}{y_1} \qquad \text{(III.5b)}$$

The notations are similar to the notations that we use in our Lanchester analysis, and x_1 and y_1 are the respective staying powers of the ships. Looking at Equations III.5a and III.5b, and comparing it to Lanchester's own equations, Equations III.1a and III.1b, we see that Lanchester has an implicit assumption that

[3] Staying Power

the staying power of the unit is 1. By explicitly expressing the staying power[4] of a ship, Chase was able to make the following conclusion: [Ref. 12].

- If there are twice as many units on one side as on the other, for parity, each unit of the force with the smaller number of units must be twice as strong in fighting power and twice as strong in staying power.

Both Lanchester and Chase made the assumption that there is an exchange of continuous fire in a battle. This would accurately reflect the battle conditions then. Both models measured victory by attrition. The model used in the next chapter is strongly related to these concepts, but explicitly expresses staying power and defensive power as part of the force effectiveness calculations. It also updates the concepts to ensure it applicability to modern naval combat. The other significant difference is that the next model will not be for continuous fire. Rather it is a “pulsed” or “salvo” model.

[4] Chase defines this as the defensive power

IV. THE NAVAL SALVO MODEL

> *"Shall we construct from these materials* **one** *or* **two** *ships? ...if we decide to build* **one** *instead of* **two***, then this* **single** *ship must be* **TWICE** *as strong offensively and* **TWICE** *as strong defensively*[1] *as one of the two ships.*
>
> *It seems to me that while it may be possible to make a ship carry twice as many guns as one of half the displacement it is, at least, debatable if she can be made twice as strong defensively. The chances of hitting her are certainly much greater and she certainly is* **NOT TWICE** *as strong defensively against underwater attack."*

— RADM J.V. Chase, 1921

A. INTRODUCTION

The aim of this chapter is to analyze the effects of force distribution using the Naval Salvo Equations[2] as a baseline for all calculations.

1. Organization

The first section introduces the basic form of the Salvo Equations. The variables, terminology, and assumptions of the equations will be presented. We will discuss, in particular, the implicit assumptions associated with the deterministic form of the equations, and one key variable in the analysis, the *staying power* of a ship. Using the Naval Salvo Equations, we will discuss the military worth of distribution of a fleet in a missile exchange.

The next section will focus on re-addressing some of the assumptions of the equations and to use the deterministic equations as a basis for the development of a simple stochastic model. In this section, we will discuss the effects of distribution from a purely defensive point of view. We will discuss the concept of "instability" when

[1]When Chase referred to defensive, he was referring to the staying power.

[2]The Naval Salvo Model was originally developed by CAPT. Wayne Hughes, Naval Postgraduate School.

a fleet has an unbalanced distribution of power. And we will discuss the conditions when force concentration is preferable to force distribution.

Following which, we will develop the probabilistic model further to incorporate more realism into the analysis. The model used in this section is a stochastic, Salvo Equation-based model. The model was initially developed by John McGunnigle in his thesis, *An Exploratory Analysis of the Military Value of Information and Force* [Ref. 16]. We will use this model to discuss the effects of force distribution.

In the final section of this chapter, we will present results obtained from campaign analyses performed by students of the Naval Postgraduate School.

B. BASIC SALVO EQUATIONS

The Salvo Equations and its mechanics are covered in detail in *Fleet Tactics and Coastal Combat* [Ref. 9]. The basic pair of equations are as follows:

$$\Delta A = \frac{\beta B - a_3 A}{a_1} \tag{IV.1a}$$

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \tag{IV.1b}$$

Where,

- A = number of units in force A;
- B = number of units in force B;
- α = number of well aimed missiles fired by each A unit;
- β = number of well aimed missiles fired by each B unit;
- a_1 = number of hits by B's missiles needed to put one A unit out of action;
- b_1 = number of hits by A's missiles needed to put one B unit out of action;
- a_3 = number of well-aimed missiles destroyed by each A;
- b_3 = number of well-aimed missiles destroyed by each B;
- ΔA= number of units in force A out of action from B's Salvo;
- ΔB= number of units in force A out of action from A's Salvo.

1. Terminology Used in the Naval Salvo Model

The term, *out of action*, means that the ship cannot operationally contribute to the mission any longer. Its assets (e.g., missiles, radars, etc.) cannot be used any longer, but it may still remain afloat. Definitions of other terms may be found in *Value of Warship Attributes in Missile Combat* [Ref. 12].

2. Assumptions of the Basic Naval Salvo Equations

There are many assumptions associated with the use of the equations, and they are covered in detail by CAPT. Wayne Hughes in *Value of Warship Attributes in Missile Combat* [Ref. 12]. We will discuss the assumptions that have relevance to our study. The following assumptions are true when both equations are used in a deterministic manner:

- Both A and B are able to see, and fire upon each and every ship of its adversary.
- There is no wastage of missiles as long as there are targets. Every well aimed missile will hit a target. If there are no targets available, the fired missile is deemed an overkill.

The first assumption offers a very important insight, the importance of striking effectively first.

> *"I have found again and again, that in encounter actions, the day goes to the side that is the first to plaster its opponents with fire. The man who lies low and awaits developments usually comes off second best [Ref. 15]."*

— Generalfeldmarschall Erwin Rommel

3. Discussion of Staying Power, b_1

One of the cornerstones of using the Salvo Equations for analysis is the **staying power** of a ship—the number of missiles that is required to put a ship out of action. It is represented by the subscript 1, (e.g., b_1 and a_1). It is important to note that, based on empirical evidence; the staying power of a ship is not a linear function of the displacement of the ship. As a ship grows in size, its staying power does not grow linearly with its size but approaches a maximum value for ships beyond a certain size.

There have been many studies performed to calculate the amount of ordnance required to put a ship out of action, prominent among which are: *Beall* [Ref. 2], *Humphrey* [Ref. 13], and *Schulte* [Ref. 19]. A summary of these findings are presented in *Fleet Tactics and Coastal Combat* [Ref. 9]. Based on empirical evidence, the findings conclude that the staying power of a ship is approximately proportional to its length, rather than its volume. We will illustrate this by estimating the number of missiles required to put a 90,000 ton combatant out of action.

4. Estimated Staying Power of Modern Surface Combatants

Chapter 6 of *Fleet Tactics and Coastal Combat* [Ref. 9], shows (with empirical evidence) that a ship which displaces 7000 tons or less, requires approximately a single hit from an *Exocet* missile to be put out of action. The results for larger crafts are still classified at the moment. If we assume that the number of missiles required to put a ship out of action is proportional to its length, we can estimate the number of missiles required to put a 90,000 ton ship out of action.

$$
\begin{aligned}
&= \frac{\sqrt[\frac{1}{3}]{90000}}{\sqrt[\frac{1}{3}]{7000}} \times 1 \\
&\approx 2.3
\end{aligned}
$$

This equation shows that it requires approximately 2.3 missiles to put a 90,000 ton combatant out of action. This phenomenon poses a dilemma for naval planners: Is it advantageous to build a 90,000 ton ship, with a staying power of 3 missiles that carries 100 times the firepower of a 900 ton ship that has a staying power of 1 missile? Or would it make more military sense to build 100 of the 900 ton ship, with an aggregate staying power of 100 missiles? From B's defensive point of view, an enemy firing a salvo of 100 missiles to destroy the fleet has to get just 3 good shots to win the battle. Whereas, if he had 100 of his 900-ton combatants, the enemy would have to get all 100 good shots through the defenses to achieve the same result. Figure 11 illustrates the consequences of power concentration. Each small circle represents the CEP of a single missile.

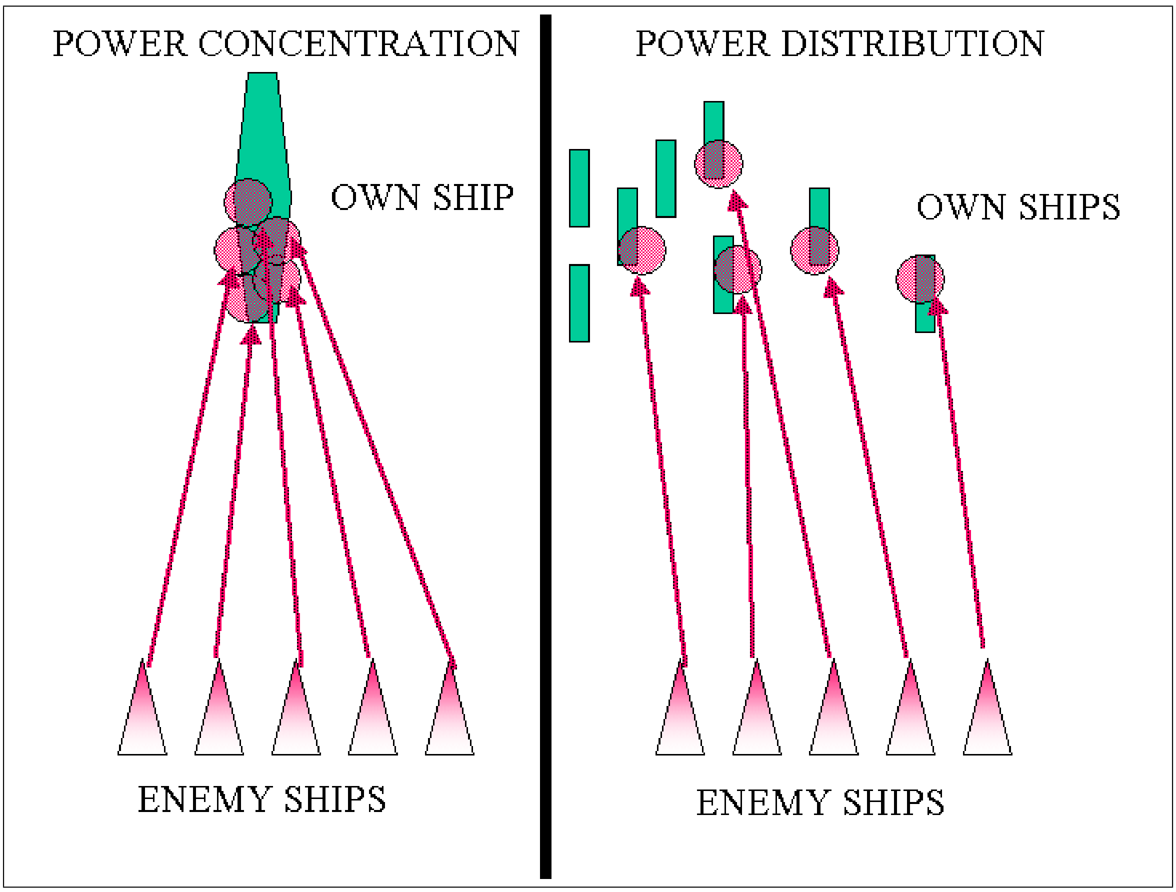


Figure 11. Effects of Distribution on Survivability

C. THE MILITARY WORTH OF DISTRIBUTION USING DETERMINISTIC SALVO ANALYSIS

Equations IV.1a and IV.1b measure the actual number of ships lost. In most circumstances, a fractional loss or survival rate is more revealing. For example, a loss of 5 ships will affect a 10-ship fleet more than it would affect a 300-ship fleet. We will modify the two equations to measure the fractional loss of each side.

$$\begin{aligned}\frac{\Delta A}{A} &= \frac{\beta B - a_3 A}{A \times a_1} \\ \frac{\Delta B}{B} &= \frac{\alpha A - b_3 B}{B \times b_1}\end{aligned} \tag{IV.2}$$

A fair comparison must be made between a distributed force and a concentrated force. We shall assume the following in our comparison.

- The total offensive power and defensive power of a fleet are kept constant. This implies that βB (offensive power) and $b_3 B$ (defensive power) are kept constant throughout. Implicit to this assumption is that as B chooses to use fewer platforms, he will concentrate more offensive and defensive firepower per ship.
- As more defensive and offensive firepower are added to a ship, the size and staying power increases. **BUT**, the staying power of a ship, b_1, does not increase in proportion to its increase in firepower. The relationship is given in Table III.

The total amount of defensive and offensive firepower of all configurations is kept constant. Table III tabulates the parameters for a fleet with a total defensive power of 40 and total offensive power of 50.

From Table III, it can be seen that if B opts to have fewer ships in his fleet, each of his ship will carry more offensive and defensive firepower (Assumption 1). If he chooses to use a more distributed fleet, each ship will have significantly less offensive and defensive power. For example if B chooses to have only 2 ships in his fleet, each ship will carry 20 units of defensive power, 25 units of offensive power and have a staying power of 3 units.

By imposing these conditions and looking at Equation IV.2, we see that the numerator of the fractional loss for B[3] is constant. The physical interpretation of the numerator is the number of *leakers*. We will examine the case for 1, 2, and 3 leakers. Additionally, we see that the whole term for the fractional loss for A is also

[3] The numerator is $\alpha A - b_3 B$. α, A, $b_3 B$ are all constant.

Number of B ships	Staying Power per ship, b_1	Defensive Power per ship, b_3	Offensive Power per ship, β
1	3	40	50
2	3	20	25
3	2	13.3	16.7
4	2	10	12.5
5	1	8	10
6	1	6.7	8.3
7	1	5.7	7.1
8	1	5	6.25
9	1	4.4	5.6
10	1	4	5

Table III. Parameters for B's Fleet

a constant. Using Equation IV.2, we will only plot the graph of the relationship between the fractional loss for B ($\frac{\Delta B}{B}$) and the number of B. Results are shown in Figure 12.

1. Discussion of Results

From Figure 12, there are three lines drawn on the graph. The top line would represent the case for **3 leakers**, the second for **2 leakers** and the bottom for **1 leaker**. In general, the more leakers there are, the higher will be the fractional losses, $\frac{\Delta B}{B}$, of the fleet.

The second peculiarity about the graph is that it seems to show that as we move from 2 ships to 3 ships, there seems to be no effect on the survivability of the fleet. Looking at Table III, it can be seen that for these two particular instances (2 ships and 3 ships), the aggregated staying power is the same, 6 in both cases. One of the key assumptions in the Basic Naval Salvo Model is that the capabilities of a ship are proportionally degraded with each hit that the ship sustains. Therefore, since the aggregated staying power in these two cases are the same, each leaker will cause the same amount of degradation in both cases. Hence, there seems to be no effects when

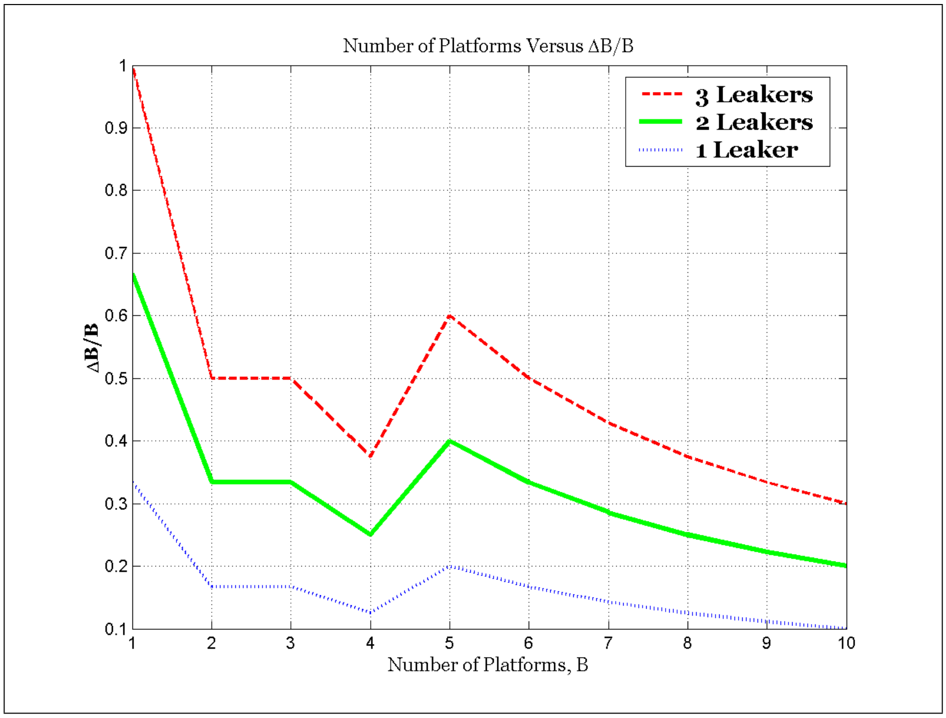


Figure 12. Effects of Distribution on Fractional Loss

the fleet is increased from a 2 ship configuration to a 3 ship configuration.

The third trend to note is that as more and more platforms are used, the fractional losses decrease. There seems to be a discontinuity as the number of platforms used changes from 4 to 5. The reason for the spike at B=5, is that from B=5 onwards, the fleet is getting bigger in numbers, but each ship now has a reduced staying power of only 1 missile. Consider the case when there are 3 leakers: B would have lost 60% of his fleet if he had chosen the 5-ship configuration. If he had chosen the 4-ship configuration, he would only have lost about 37.5% of his fleet. But if he had chosen the 10-ship configuration, his losses would only have been 30%. This implies that there might be a threshold that must be exceeded before distribution becomes an attractive option. Consider the case when there are 3 leakers: From Figure 12, when B chooses to have 4 platforms, he suffers a loss of about 38% in the salvo. If he had opted for a distributed fleet, he would not have achieved this result until he had about 8 platforms. This is due to the **aggregated staying power** of the fleet.

When he has 4 ships, each with a staying power of 2 missiles, the fleet has a total aggregated staying power of 8 missiles. When he has 8 ships, each with a unit staying power, the aggregated staying power is also 8. Therefore it is only attractive, from a defensive point of view, to distribute when the aggregated staying power of the distributed fleet exceeds the aggregated staying power of the concentrated fleet.

Since we have assumed that firepower is equally distributed among a fleet, if 30% of the total fleet is put out of action, then it would mean that 30% of the total fleet firepower is also lost. Therefore the trend shows that with a fixed amount of leakers, a fleet will lose more percentage of its firepower in a single enemy salvo if it were a concentrated fleet. As the fleet gets more distributed, provided it has crossed a certain "minimum" or threshold value for distribution, the percentage of firepower lost per enemy salvo decreases.

As the number of leakers increases from 1 to 3, the reduction in losses is more pronounced for a distributed force. Consider the case when there is just 1 leaker. If B has 5 ships, each with a unit staying power, his losses after the first salvo is 1 ship. If he had distributed his fleet to 10 ships, his losses would have been 1 ship. But since we have distributed the firepower of the ships, in the first instance, B would have lost 20% of his firepower, whereas in the second case, only 10%. We can conclude that by distributing a force, we have a much better chance of graceful degradation. From another point of view, it also means that as compared to a concentrated fleet, a distributed fleet is capable of retaining more of its combat potential after sustaining reasonable losses, and will have a higher probability of mission success in such circumstances. Distribution makes a fleet more robust.

It is obvious that if a fleet is made up of big powerful ships, the percentage of **firepower lost per leaker** is very much greater. For our simple case, we have tabulated the firepower lost per leaker in Table IV. Take the case of 1 ship, if 3 leakers are present, the percentage of firepower lost is 100%, because the ship is put out of action. This implies that 1 leaker destroys 33% of the total fleet firepower. If we have 2 ships, and there are 3 leakers, the fleet will be reduced in capability as it will only 1 ship left, i.e., 50%. Therefore, one leaker will effectively destroy $\frac{50}{3}\%$ of the total fleet's firepower.

Number of B ships	Staying Power per ship, b_1	% of Fleet Firepower lost Per Leaker
1	3	33%
2	3	16.7%
3	2	16.7%
4	2	12.5%
5	1	20%
6	1	16.7%
7	1	14.3%
8	1	12.5%
9	1	11.1%
10	1	10%

Table IV. Fleet Firepower Lost Per Leaker

Looking at it from another point of view, consider the case when B has 2 ships, each with an offensive power of 25 missiles, and a defensive power of 20 missiles, and a staying power of 3 missiles. Consider the case when there are 3 leakers. B would have lost one ship, and along with it, 25 offensive missiles and 20 defensive missiles—(50% of his initial firepower)— and the total number of leakers required to put the entire fleet out of action now has been reduced to 3. Now consider the case when B has a distributed force of 10 ships. Consider when there are 3 leakers. B would have lost 3 of his ships, but he would only have lost 15 offensive missiles and 12 defensive missiles —he has only lost 30% of his fleet firepower. He still has a striking power of 35 missiles, and 28 defensive missiles. And the total number of leakers that is required to defeat B is 7 leakers.

In this example, because staying power is not a linear function, we see that the 2-ship fleet had an aggregated staying power of only 6 missiles. Which means the enemy only needs 6 leakers to win the battle. To win the battle against the distributed fleet, the enemy needs to have 10 leakers to penetrate the defense.

2. Conclusions From Basic Deterministic Salvo Analysis

Distribution decreases the fractional loss of a fleet and allows the fleet to maintain a significant proportion of its original striking power even after sustaining some damage. In comparison, a highly concentrated fleet loses a significant percentage of its properties after sustaining damage. These trends highlight the fact that when a ship is put out of action in a missile exchange, it (and the whole fleet) loses all its properties (defensive, offensive and staying power). And if a fleet chooses to concentrate its power in a few ships, the losses that accrue with every leaker is more catastrophic as compared to a distributed fleet. This is situation referred to as "instability" in a paper by CAPT Hughes [Ref. 12]. From our analysis, this can be measured in the percentage of fleet firepower lost per leaker. The higher this value is, the more "instability" there is in a fleet.

When a fleet is distributed, the enemy will require to expend more offensive missiles to put the fleet out of action. Even if the total number of defensive missiles is kept constant, distribution forces the enemy to find and target more ships and in the process, expend more offensive missiles. This occurs because distribution increases the aggregate staying power of a fleet. From a trade-off point of view, distribution "purchases" more staying power for a fleet. This is literally true, because more offensive and defensive power, BUT not staying power, per dollar of construction can be built into big ships rather than small ones.

On the other hand, distribution is only attractive when the sum of staying powers of a distributed fleet exceeds the sum of staying powers of a concentrated fleet. Another implication of this phenomenon is that when a distributed fleet is employed, it must be employed in its totality to be more attractive than a concentrated fleet.

D. SIMPLE STOCHASTIC SALVO MODEL

We now address the two assumptions about the deterministic Salvo Analysis. First let us assume that an enemy does not have perfect knowledge about the status of the fleet. And because of this, the enemy randomly targets the ships of the opposing fleet. We will have to bring the deterministic Salvo Analysis to a higher resolution.

1. Process Modeling

The model revolves around ten ships. Each ship is assigned a military worth (independent of its fighting characteristics). Each vessel is also given a defensive power and a staying power that the Naval Salvo model describes. The aim of this model is to investigate if distribution of military worth among a fleet of ships will sustain less damage (loss of military worth) than a fleet which has its military worth concentrated in a few ships. This military worth may be translated as offensive firepower on board a single ship or it may even be viewed as cost of the ship.

2. Simple Stochastic Salvo Assumptions

The assumptions associated with this model are as follows:

- The enemy is not described in ships. Instead the enemy is a salvo of 60 missiles;
- Each ship has an equal probability of being targeted by the enemy;
- The military worth of the ship is reduced linearly in proportion to its staying power. If a ship has a military worth of 10 units, and a staying power of 2 missiles, a single missile hit on this ship would reduce the military worth to 5 units;
- If a kill or overkill occurs, the ship is assumed out of action and has its military worth reduced to 0;
- The staying power of a ship is NOT linear with respect to its military worth. The relationship between staying power and military worth are tabulated in Table V. For example, if a ship has a military worth of 20 units, its associated staying power will be 3 missiles. If a ship has a military worth of only 6 units, its staying power is only 2 missiles[4]. This is a reasonable approximation to

[4] Note that in the case where the military worth of a ship is 2.5, the staying power is 1.

reality because if a ship is considered a high value asset, it is only logical that it would have been constructed to withstand more damage.

Refer to Appendix B for a numerical example illustrating the enemy's targeting process and the fractional loss calculation.

Military Worth	Staying Power, b_1
10 and above	3
9	2
8	2
7	2
6	2
5	2
4	1
3	1
2	1
1	1

Table V. Parameters for the Fleet

We will impose our two conditions mentioned earlier. The fleet's total military value remains constant at 50 military worth units and its total defensive firepower is constant at 40 missiles. We will investigate 7 different configurations, ranging from a highly concentrated fleet to a very distributed fleet. The 7 configurations are listed in Table VI.

For purposes of graphical clarity, Figure 13 is provided as an example. In this simple example, there were **10** engagements simulated for each of the 7 configurations. The enemy was given a total of 40 missiles to launch at B.

- Each marker represents a single encounter;
- Each distribution pattern is run for 10 encounters;
- The legend [25 25] means B has 2 ships, each worth 25 units
- The legend 10 x 5 means that B has 10 ships, each worth 5 units

Configuration	Number of ships	Value per ship	Defensive Power per ship	Staying Power per ship	Representation
1	2	25	20	3	CIRCLES
2	3	16.7	13.3	3	STARS
3	5	10	8	3	SQUARES
4	10	5	4	2	DIAMONDS
5	20	2.5	2	1	ASTERISK
6	25	2	1.6	1	STAR OF DAVID
7	50	1	0.8	1	TRIANGLE

Table VI. Parameters for 7 Different Configurations

- Each distribution pattern is spread over a unit on the x-axis for purposes of clarity. For example, the entire first column represents a configuration of 2 ships, each worth 25 units. Looking at this column, out of the 10 encounters, B survived 3 encounters intact. B lost about 0.17 of his fleet value in 2 encounters, about 0.35 in 1 encounter and 0.5 in 4 encounters. Each symbol represents a different configuration and each configuration spans a unit width on the x-axis.

3. Results From Simple Stochastic Analysis

We now run the model for 100 encounters for each configuration. We will also investigate how distribution is affected when the enemy's firepower is varied. A single graph will show how different configurations affect fractional losses for a particular enemy strength. For example, Figure 14 shows the fractional losses for each of the 7 configurations when the enemy launches 60 incoming missiles in a single salvo. Figure 15 shows the results when there the enemy launches 50 incoming missiles. The enemy strength is varied from 60 missiles to 10 missiles and the results are shown in Figures 14 to 19. The mean and standard deviations of the data may be found in Table VII and Table VIII respectively. Enlarged graphs are attached as Appendix C.

Referring to the results of Figure 14, when B has a configuration of 2 ships each

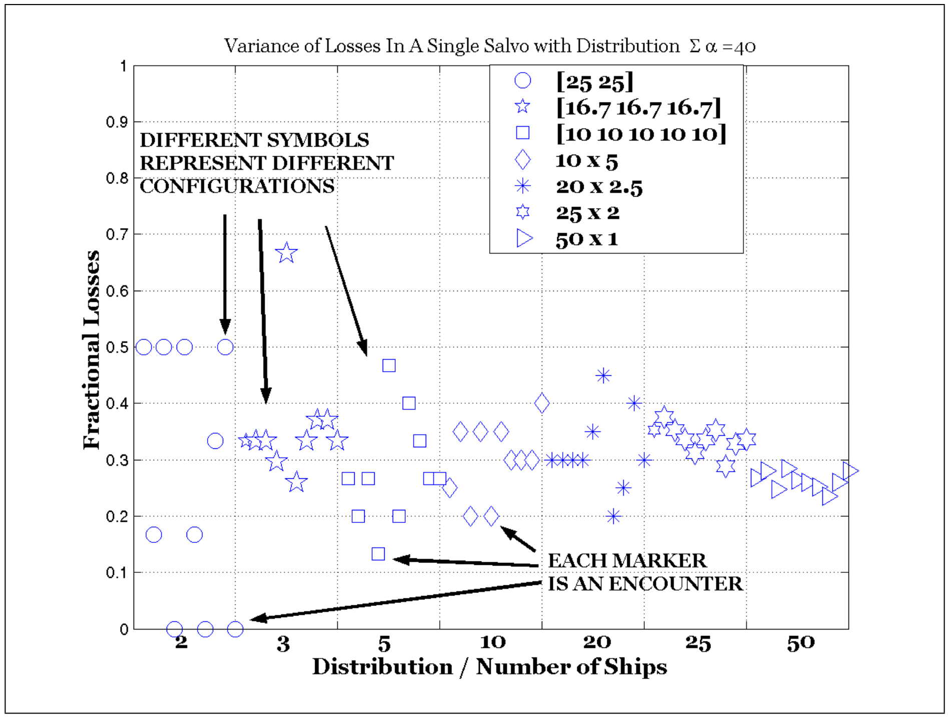


Figure 13. Example of Variance of Losses in a Single Salvo, 10 runs, Enemy Firepower = 40

worth 25 units (the circles in the first column of the graph), he loses 100% of his assets in most of the 100 encounters, only managing to survive (though heavily attrited) in 4 instances out of the 100 encounters. When he has 3 ships, each worth 16.7 units, his losses are more spread out, ranging from 65% to 100%. As he distributes his forces from 2 ships to 50 ships, his loses are reduced and the spread of his losses are also reduced. Referring to the first column of Table VII, the mean losses generally decrease as he distributes his fleet. Looking at the standard deviation of his losses in the first column of Table VIII, it can be seen that as he distributes his fleet, the range of his losses (indicated by the standard deviation) decreases. This implies that as he distributes his forces, his losses become more predictable and more stable.

Now let us compare our results with the deterministic Salvo Equations. When the total number of enemy missiles is 60, and B has chosen to deploy 2 ships, each worth 25 units:

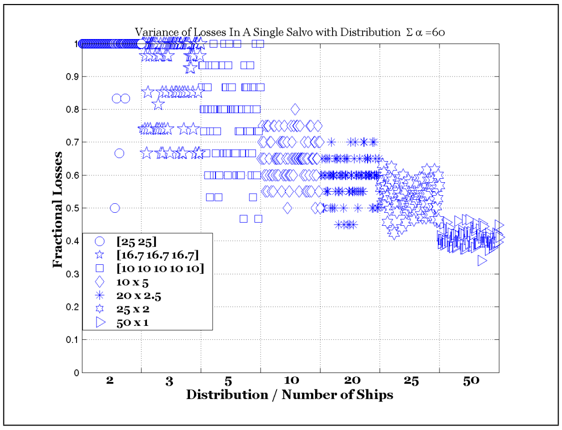


Figure 14. Fractional Loss with 60 Enemy Missiles

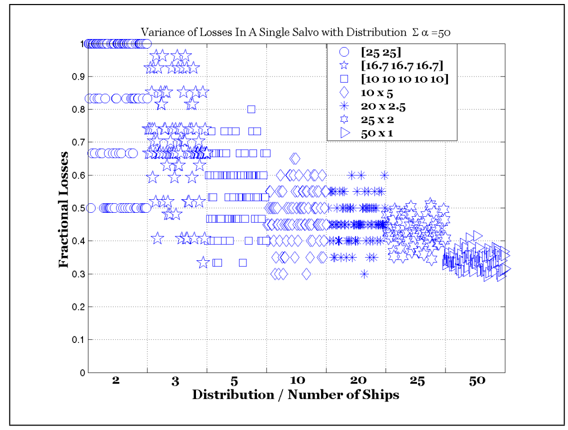


Figure 15. Fractional Loss with 50 Enemy Missiles

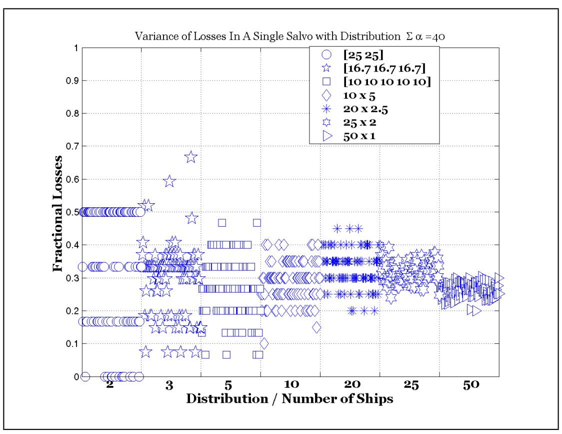


Figure 16. Fractional Loss with 40 Enemy Missiles

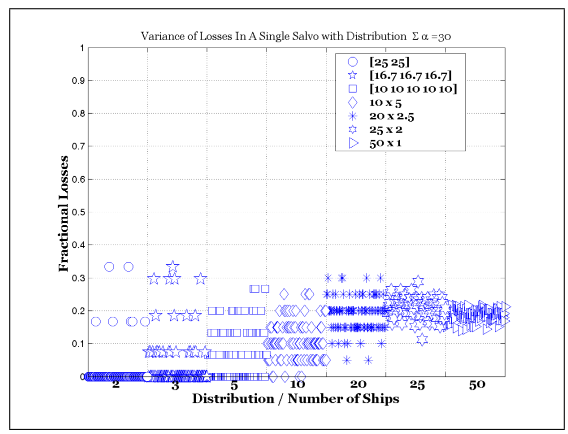
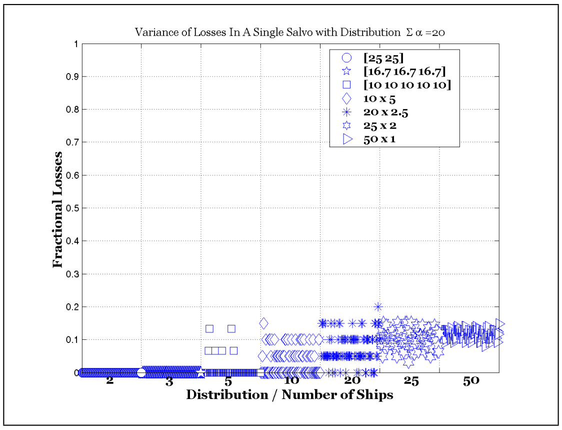


Figure 17. Fractional Loss with 30 Enemy Missiles

$$\frac{\Delta B}{B} = \frac{\alpha A - b_3 B}{B \times b_1}$$
$$\frac{\Delta B}{B} = \frac{60 - 40}{2 \times 3}$$
$$\frac{\Delta B}{B} = 3.33$$

A has the ability to defeat B with more than 200% overkill. From Figure 14, results indicate that with this configuration, B loses in almost all the 100 encounters. Now consider when B has chosen Configuration 3, with 5 platforms. Substituting into the fractional loss salvo equation,



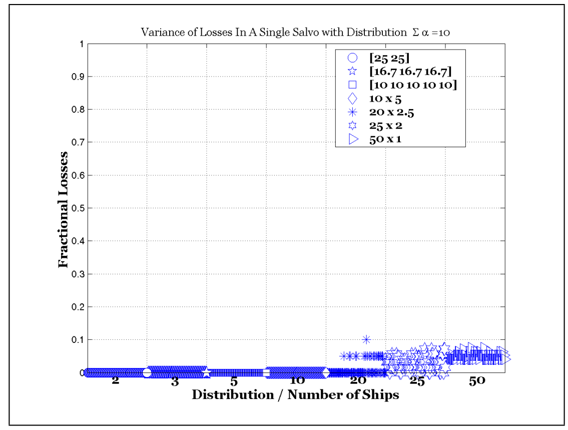


Figure 18. Fractional Loss with 20 Enemy Missiles

Figure 19. Fractional Loss with 10 Enemy Missiles

Number of ships	60 Enemy Missiles	50 Enemy Missiles	40 Enemy Missiles	30 Enemy Missiles	20 Enemy Missiles	10 Enemy Missiles
2	0.99	0.85	0.34	0.02	0	0
3	0.91	0.70	0.33	0.06	0	0
5	0.78	0.55	0.27	0.08	0	0
10	0.64	0.49	0.29	0.12	0.03	0
20	0.58	0.46	0.34	0.19	0.08	0.01
25	0.54	0.44	0.32	0.20	0.1	0.02
50	0.42	0.34	0.26	0.19	0.1	0.05

Table VII. Mean of Results From Simple Stochastic Model

$$\frac{\Delta B}{B} = \frac{\alpha A - b_3 B}{B \times b_1}$$
$$\frac{\Delta B}{B} = \frac{60 - 40}{5 \times 3}$$
$$\frac{\Delta B}{B} = 1.33$$

A has the ability to defeat B with about 33% overkill. But with this configuration (Configuration 3), results in Figure 14 indicate that the fractional loss of B is less than that calculated with the deterministic Salvo Equations. The losses with Configuration 3 ranges from 100% to about 45%, with the mean about 75%. This occurs because the enemy does not equally distribute his missiles among the targets.

Number of ships	60 Enemy Missiles	50 Enemy Missiles	40 Enemy Missiles	30 Enemy Missiles	20 Enemy Missiles	10 Enemy Missiles
2	0.05	0.20	0.16	0.07	0	0
3	0.12	0.12	0.11	0.09	0	0
5	0.12	0.10	0.09	0.07	0.02	0
10	0.08	0.08	0.07	0.07	0.03	0
20	0.06	0.06	0.06	0.05	0.05	0.02
25	0.04	0.03	0.03	0.03	0.03	0.02
50	0.03	0.03	0.02	0.01	0.01	0.01

Table VIII. Standard Deviation of Results From Simple Stochastic Model

For the simple stochastic model to agree with the deterministic model, the enemy would have to uniformly distribute his missiles against B's fleet. However, the enemy is neither unaware of the number of missiles that would be required to put one of B's ships out of action, nor is he aware of the number of defensive missiles that B has. Therefore, the allocation of missiles against B would be more random than uniform. This arbitrary allocation of missiles causes overkill in some cases and "underkill[5]" in others. When overkill occurs, there will be fewer missiles that is available to be directed against another ship of B's fleet. It is because of this overkill, which is modeled into the simple stochastic model, that allows the fleet to survive with about 75% losses rather than a total defeat as calculated by the deterministic Salvo Equations.

As B increases his fleet numbers, the probability of "inefficient" targeting by the enemy increases. With fewer platforms, the probability of overkill is very high. As the numbers increase, the probability of overkill decreases and the probability of underkill increases. Therefore results indicate that distribution increases the survivability of the fleet.

In Figure 14, we also observe that the range of results gets narrower[6] as the distribution increases. The effects are still more pronounced in Figure 15.

[5]Underkill occurs when a salvo launched in unable to put the whole ship out of action

[6]"Instability" decreases as the fleet gets more distributed OR has a lower percentage of total military worth lost per leaker.

As we decrease the enemy's capability, the results display another trend. Consider Figure 15: As the enemy's capability is reduced from 60 to 50 missiles, the 2-ship fleet has managed to survive more often (as compared to when the enemy's capability was 60 missiles) in the 100 encounters that were simulated. A similar trend is observed for the 3-ship fleet. In Figure 15, there appears to be more instances of the concentrated fleet surviving as the enemy's capability is decreased. Figures 16 to 19 illustrate the same trend.

Figure 16 shows the results when the number of missiles that the enemy launches is exactly equal to the number of missiles that B can shoot down. From the deterministic Salvo calculation,

$$\frac{\Delta B}{B} = \frac{\alpha A - b_3 B}{B \times b_1}$$
$$\frac{\Delta B}{B} = \frac{40 - 40}{2 \times 3}$$
$$\frac{\Delta B}{B} = 0$$

It shows that when this happens, B should have suffered no losses as he has managed to shoot down all the incoming missiles. But results from Figure 16 show that B still sustains losses in this instance. This is due to the randomness of the enemy's allocation as explained earlier. It is because of this arbitrary targeting of B that allows A to inflict some damage on B. In this case, B's ships are all capable of shooting down 4 incoming missiles each, but because of the random targeting, A will target some platforms of B with 4 or less missiles (in which case that particular platform will survive), and in other cases more than 4 (in which case B sustains some hits).

Now consider when the enemy's missiles are further lowered to 30 missiles in Figure 17. In this case, distribution no longer becomes an attractive option. As the fleet becomes more distributed, the fractional losses incurred in a single salvo increases. Looking at Figure 18 when the enemy launches a total of 20 missiles against

B. In this instance when B is concentrated and has a 2-ship fleet, each of his ships is able to defeat 20 incoming missiles. Therefore, no matter how the enemy distributes his missiles against the 2 ships of B, B is able to shoot down all the incoming missiles. Even in the worst case scenario, where the enemy targets all his 20 missiles against one single ship of B's, B still manages to shoot down all of the incoming missiles. B is almost invulnerable in this configuration. When B distributes his fleet in this scenario to 10 ships, each individual ship can only defeat 4 incoming missiles. Therefore, in cases where A targets a single platform of B with more than 4 missiles, B will sustain some hits. In general, when the enemy's offensive power is greater than one's own defensive power, distribution is an effective option, but when one's own defensive power is much greater than the enemy's offensive, then distribution will no longer be as effective as it was.

4. Conclusions From Simple Stochastic Salvo Analysis

Massing of assets for defense is attractive when the total defensive firepower of a fleet exceeds the total offensive firepower of the enemy. When the enemy's offensive is strong, and our forces have weak defenses, it is more attractive to disperse, not only for defense, but doing so will also exhaust the enemy's offensive missile supplies, without sustaining much losses to our own military worth.

Denial of information and intelligence to the enemy contributes to the force effectiveness of the fleet. Without information and intelligence, the enemy is unable to efficiently allocate missiles to targets, resulting in many instances of overkills and underkills. Such instances improve the survivability of a fleet. And lastly, power concentration leads to instability in the fleet.

E. STOCHASTIC SALVO MODEL

This section will build on John McGunnigle's Thesis [Ref. 16] and the Stochastic Salvo Model that he constructed. We have reconstructed the model using MATLAB. The algorithm is attached as Appendix D. Using this model, we will investigate the effects of distribution of forces on the winning percentage and fraction of forces which survive.

In John McGunnigle's Thesis [Ref. 16], he generates his simulations for a fixed number of salvo exchanges. The model is run for 1, 2, 3 and 4 salvo exchanges. We remove that condition and modify the model to carry on with the salvo exchange until one side has been totally annihilated.

1. Intelligence Factor

The model has been modified to incorporate an "*intelligence factor*". The "*intelligence factor*" is the fraction of the enemy's ships that our forces are able to obtain information about. For example, if A's *intelligence factor* is 0.5, it means that A is able to know the status of 50% of B's fleet. A is interested only in delivering missiles to those ships of B which are still operational, but he can only know the status of 50% of B's ships. For A, those ships of which he does not know the status, he assumes that that they are still operational, and will continue to target those ships, which results in a waste of missiles. This factor is held constant for both sides throughout all salvos. See Appendix E for a detailed explanation of this parameter.

2. Stochastic Salvo Model Assumptions

The Stochastic Model has a number of assumptions associated with it, they are documented in McGunnigle's thesis [Ref. 16]. We will add on to that list.

- If a ship is known to be out of action, the enemy will not fire any more missiles at that ship. The enemy will redirect its missiles only to ships, which it assumes or knows are still operational. Each operational ship is equally likely to be targeted by enemy missiles.
- All ships with unknown status are still considered a threat, and will be fired upon.
- The fight will continue until one side has been totally destroyed.

- Each ship is assumed to have in its magazine an unlimited number of missiles and defensive missiles[7].

3. A Simple Case for Stochastic Salvo Analysis

We use a simple case to illustrate the results that the model is able to offer us. Let us use the model to generate two equal forces. Note that the missiles fired per salvo are no longer considered "well-aimed", instead, they are subjected to a probabilistic condition[8]. See McGunnigle [Ref. 16] for more details.

- A = 20 units in force A
- B = 20 units in force B
- $\alpha = 4$ aimed missiles fired by each A unit for each salvo
- $\beta = 4$ aimed missiles fired by each B unit for each salvo
- $a_1 = 1$ hit by B's missiles is needed to put one A unit out of action
- $b_1 = 1$ hit by A's missiles is needed to put one B unit out of action
- $a_3 = 5$ well-aimed missiles destroyed by each A per salvo
- $b_3 = 5$ well-aimed missiles destroyed by each B per salvo
- ΔA= number of units in force A out of action from B's Salvo
- ΔB= number of units in force B out of action from A's Salvo

For this simple case, we will assume both sides have perfect information about the opposing sides.

Figure 20 shows that if both sides are equal in terms of offensive, defensive, staying powers, and each side has the same number of combatants with the same amount of intelligence, then, in 1000 simulations, A wins almost 50% of the time, and B wins almost 50% of the time and there are very few ties.

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[7] The last assumption is to simplify the construction of the model.

[8] In fact, if the deterministic model were employed, no ships would be lost because the defensive powers, a_3 and b_3 are stronger than the offensive powers, α and β.

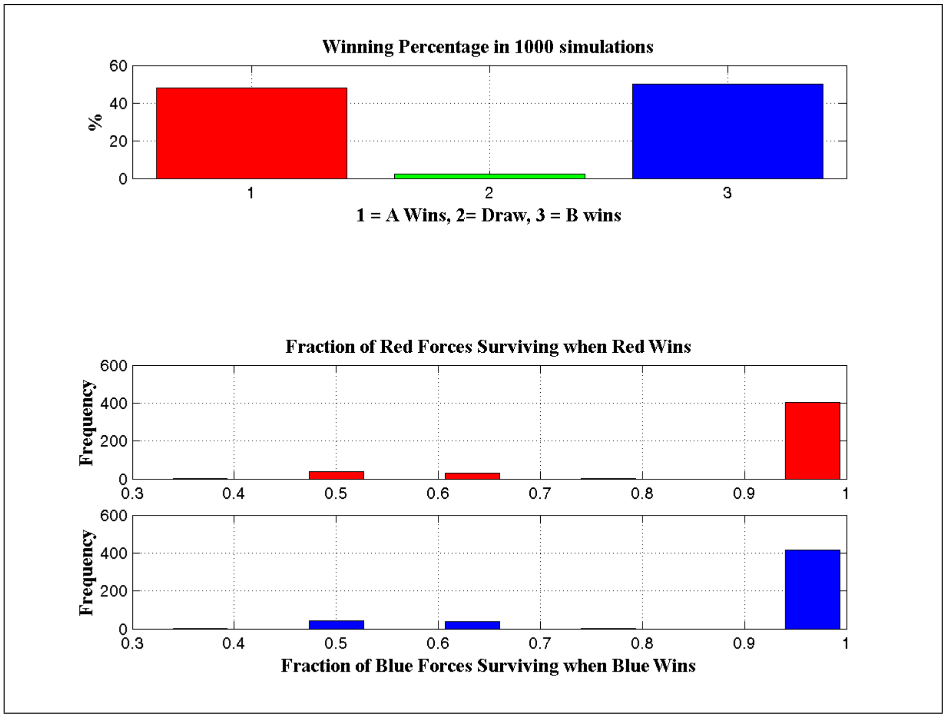

Figure 20. Stochastic Case of Equal Forces

In McGunnigle's thesis [Ref. 16], the model was simulated for 1 to 4 salvos. This model is modified to ensure that there is a clear winner. Therefore, the salvo exchange in this model will carry on until one side has been totally destroyed. This would explain the lack of draws in our results. The subplots below the top graph show the distribution of the surviving forces when a battle is over. Perhaps the most interesting feature of the subplots is that whichever side wins, most of the time the fleet manages to survive with almost all of its assets intact. Whoever is "lucky" in the first salvo exchange will go on to win with minimal losses.

4. Stochastic Salvo Analysis

In this section, we will investigate the effects of distribution of a fleet. We will investigate the effects of distributing our total assets over 1 platform to 10 platforms. As with all our previous analyses, we keep the total number of offensive missiles launched per salvo constant.

- A, Number of A forces = 15;
- B, Number of B forces, see Table IX;
- α, Firepower of A per ship per salvo = 4 missiles;
- β, Firepower of B per ship per salvo, see Table IX;
- Defensive Readiness of A = 1;
- Defensive Readiness of B = 1;
- Strike Readiness of A = 1;
- Strike Readiness of B = 1;
- a_1, Staying power of A = 1 missile hit per ship;
- b_1, Staying power of B, see Table IX;
- a_3, Defensive Firepower of A = 1 missile;
- b_3, Defensive Firepower of B, see Table IX;
- $A_{\text{intelligence factor}}$=0.8;
- $B_{\text{intelligence factor}}$=0.8[9].

The total firepower given to B is 50 and the total defensive firepower allocated to him is 40. We will run this simulation 10,000 times per configuration and find out the percentage of time that each side wins. The results are discussed in the next section.

5. Results From Stochastic Analysis

The graphical results of the experiments are shown in Figure 21 to Figure 30. The numerical results from the Stochastic Model are found in Table X.

The last two columns of the table show the average fractional force left of the **victor**.

[9]Note that when the product of the intelligence factor and the number of ships is a non-integer, the number will be rounded to the nearest integer.

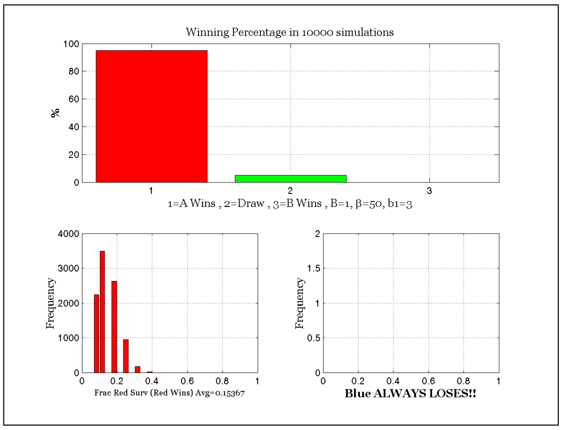

Figure 21. 1 B platform, with b_1=3

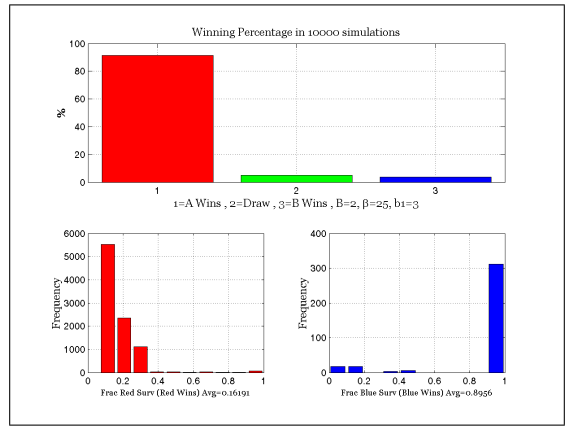

Figure 22. 2 B platforms, with b_1=3

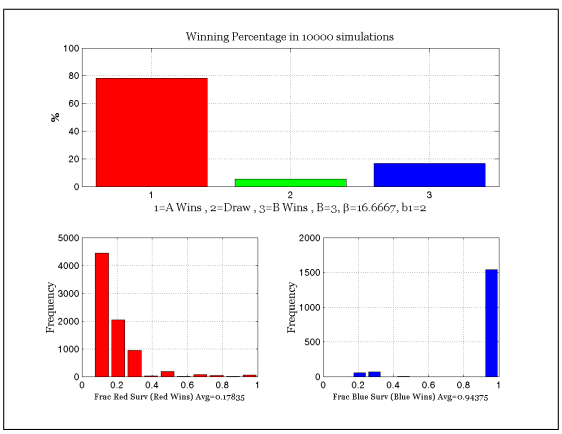

Figure 23. 3 B platforms, with b_1=2

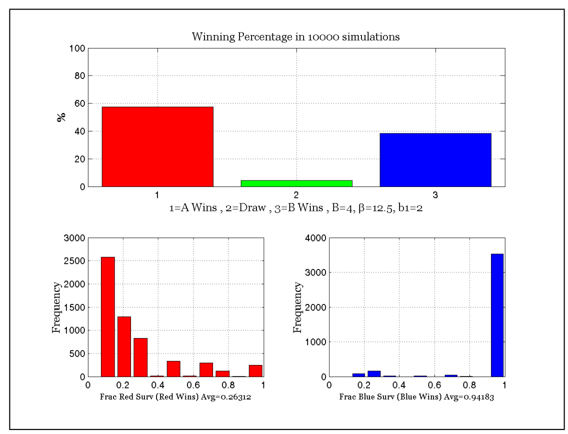

Figure 24. 4 B platforms, with b_1=2

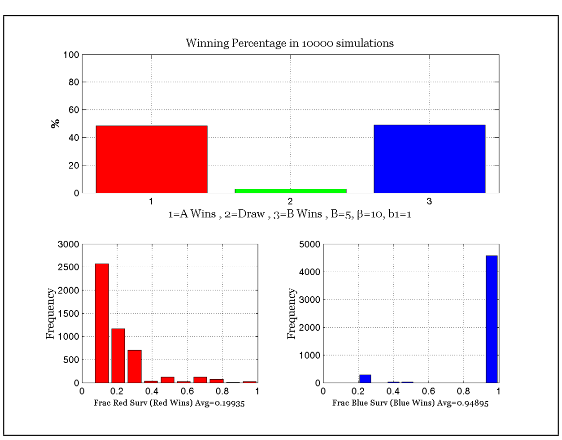

Figure 25. 5 B platforms, with b_1=1

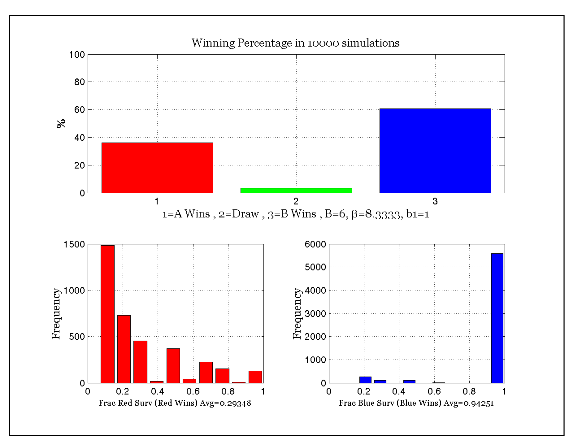

Figure 26. 6 B platforms, with b_1=1

Number of B	Offensive Firepower, β	Staying Power, b_1	Defensive Firepower, b_3
1	50	3	40
2	25	3	20
3	16.7	2	13.3
4	12.5	2	10
5	10	1	8
6	8.3	1	6.7
7	7.14	1	5.7
8	6.25	1	5
9	5.5	1	4.4
10	5	1	4

Table IX. Summary of B's Parameters(Per ship values)

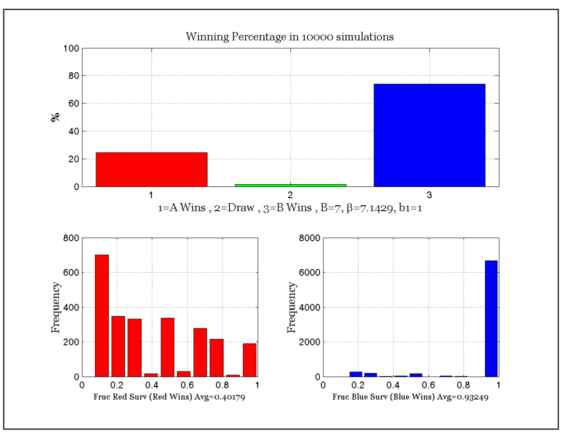

Figure 27. 7 B platforms, with b_1=1

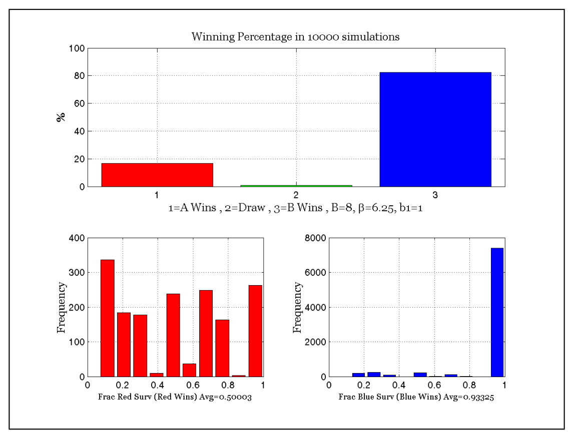

Figure 28. 8 B platforms, with b_1=1

6. Discussion of Results

Referring to Table X, it can be seen that as the firepower is distributed over a larger number of platforms, the percentage of winning has increased substantially from 0% to almost 100%. The first reason this occurs is that with a single platform, the enemy is able to concentrate his missiles (and attention) onto that platform and substantially defeat the platform as there are no other platforms to dilute the enemy's salvo of missiles, as depicted in Figure 11. As the number of platforms increases, the enemy's salvo is less concentrated and the information gained about the distributed

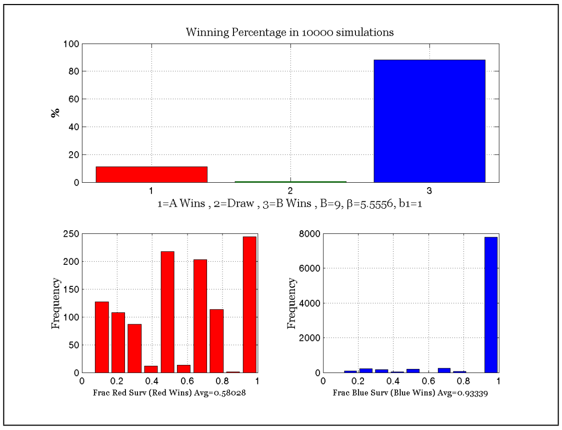

Figure 29. 9 B platforms, with b_1=1

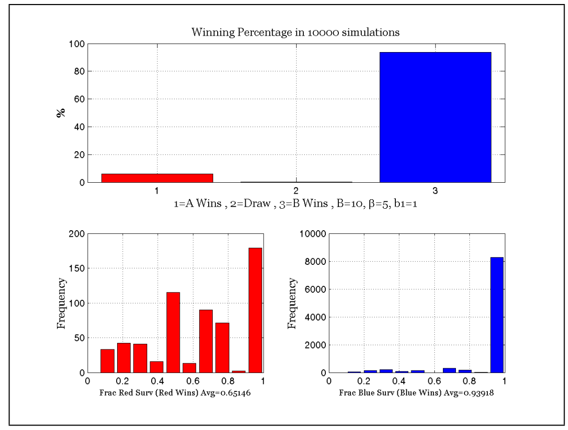

Figure 30. 10 B platforms, with b_1=1

Number of B	Win_A	$Draw$	Win_B	$Win(\%)_A$	$Win(\%)_B$	$\frac{A_{final}}{A_{initial\ avg}}$	$\frac{B_{final}}{B_{initial\ avg}}$
1	9489	511	0	94.9	0	0.15	0
2	9133	511	356	91.3	3.6	0.16	0.90
3	7809	523	1668	78.1	16.7	0.18	0.94
4	5729	425	3846	57.3	38.5	0.26	0.94
5	4834	259	4907	48.3	49.1	0.20	0.95
6	3602	325	6073	36.0	60.7	0.29	0.94
7	2453	142	7405	24.5	74.1	0.40	0.93
8	1659	91	8250	16.6	82.5	0.50	0.93
9	1125	54	8821	11.3	88.2	0.58	0.93
10	602	19	9379	6.0	93.8	0.65	0.94

Table X. Summary of Results From Stochastic Model

force is more uncertain[10]. This information (gathered by the enemy for targeting purposes) uncertainty and distribution of the fleet significantly decreases the enemy's capability to launch a concentrated or saturation attack against the fleet.

The second reason is that distribution depletes the enemy's offensive missile capability. For a large ship, it only takes 3 missiles to sink the entire "fleet" of one ship. As the force gets more distributed, no matter how small a ship is, the minimum number of missiles that must be used to destroy it cannot be less than one. Consider the following: When B has only 1 ship, the minimum number of missiles that the

[10]See the example on "intelligence factor" given in Appendix E.

enemy must fire is 43 missiles (40 to overcome the defensive missiles and 3 more to put the ship out of action). When B has 10 smaller ships, the enemy has to fire 50 missiles to ensure success (40 to overcome the defensive missiles and 10 more to put the entire fleet out of action). Therefore as B distributes his firepower, A is forced to expend more missiles to achieve the same result.

Until now we have not taken into account the depletion of missiles on board a ship. Now we extract the results (in fractional survival and depletion rates of the two sides) after 1 and 2 salvos. The results are captured in Figure 31 to Figure 40. The graphs on the top show the fractional amount of B's forces surviving after 1 and 2 salvos on the x-axis, and the fractional amount of A's forces destroyed after 1 and 2 salvos on the y-axis. The diagonal line is known as the *equity line.* Points lying on the equity line indicate that the fraction of A destroyed is equal to the fraction of B that survives. Points lying below the equity line indicate that the fraction of A destroyed is less than the fraction of B surviving. Points lying in this region are favorable to A. Points lying above the equity line are points where the fraction of A destroyed is greater than the fraction of B that have survived. Points lying in this region are favorable to B.

If missile magazine depletion is taken into account, then it might be reasonable to assume that after 1 or 2 salvos, both fleets have exhausted their missile supply. Hence, it would not be unreasonable to postulate that the status of the fleets after 1 or 2 salvos is a good indication of the eventual winner. In the first few graphs, where B is highly concentrated in a few platforms, the majority of the points lie below the equity line. If we refer to Figure 31, most of the points lie on the x-axis itself. This indicates that B has been attrited but he has failed destroy any fraction of A, a highly undesirable condition.

If we refer to Figure 33, the situation has improved for B. After the first salvo, B's fleet manages to inflict some damage on A. But if we look at the status after 2 salvos, we find that majority of the points are still lying on the x-axis and that

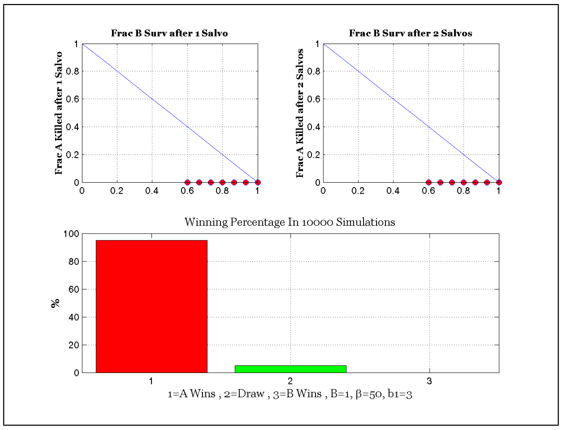

Figure 31. 1 B platform, with b_1=3, Status After 1 and 2 Salvos

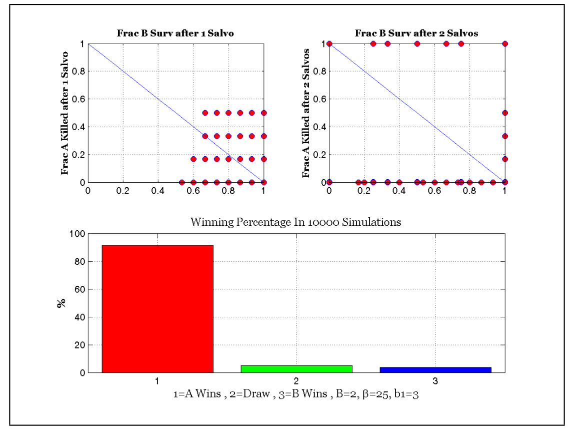

Figure 32. 2 B platforms, with b_1=3, Status After 1 and 2 Salvos

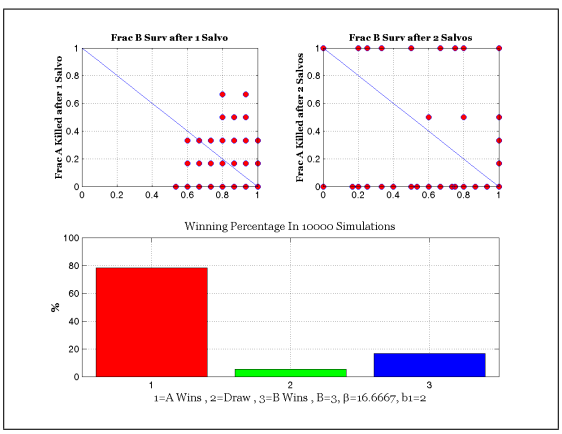

Figure 33. 3 B platforms, with b_1=2, Status After 1 and 2 Salvos

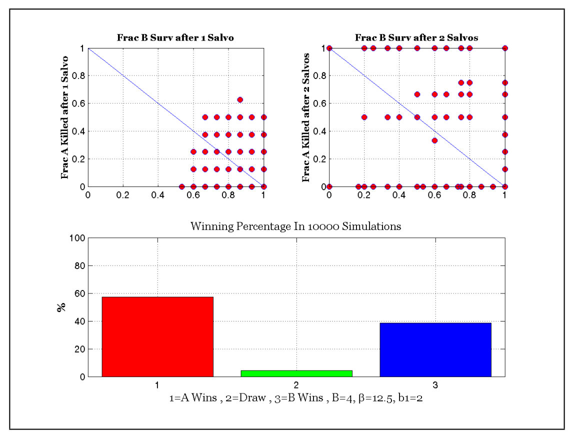

Figure 34. 4 B platforms, with b_1=2, Status After 1 and 2 Salvos

there are only a few cases where A is totally destroyed. These represent the rare cases out of 10,000 runs, where B manages to defeat A. As we track through the viewgraphs, it is clear that as B increases the distribution of his fleet, an increasing number of points are raised from the x-axis (increasing the fraction of A being put out of action). In the extreme case of distribution, Figure 40, a significant proportion of the points has been raised above the equity line. Implying that in most of the 10,000 runs, a distributed B manages to gain an advantage over A within the first 2 salvos. Comparing Figure 31 and Figure 40, the concentrated force has been outperformed by the distributed force.

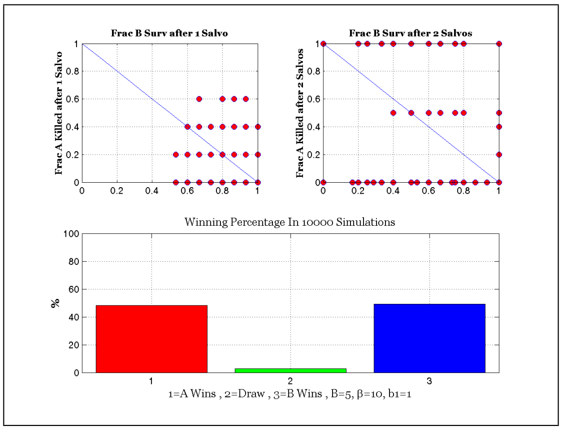

Figure 35. 5 B platforms, with b_1=1, Status After 1 and 2 Salvos

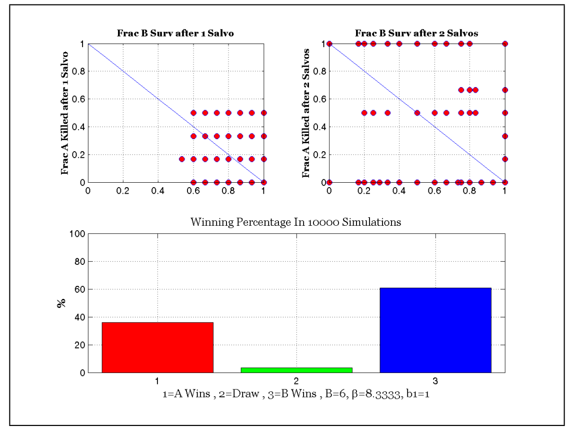

Figure 36. 6 B platforms, with b_1=1, Status After 1 and 2 Salvos

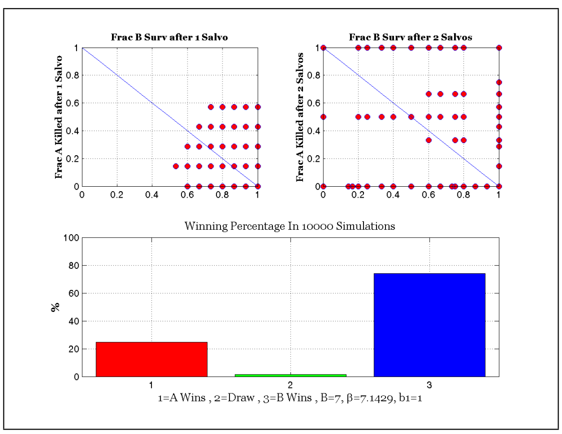

Figure 37. 7 B platforms, with b_1=1, Status After 1 and 2 Salvos

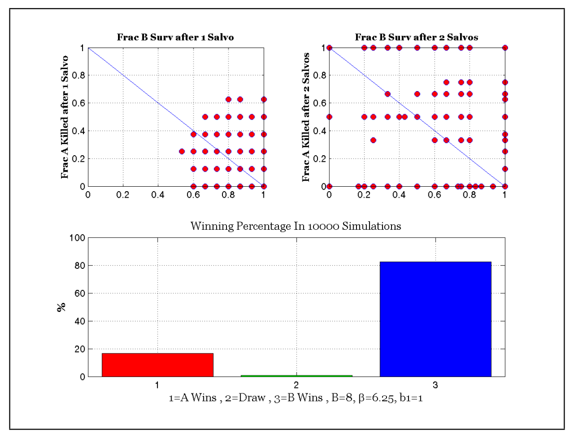

Figure 38. 8 B platforms, with b_1=1, Status After 1 and 2 Salvos

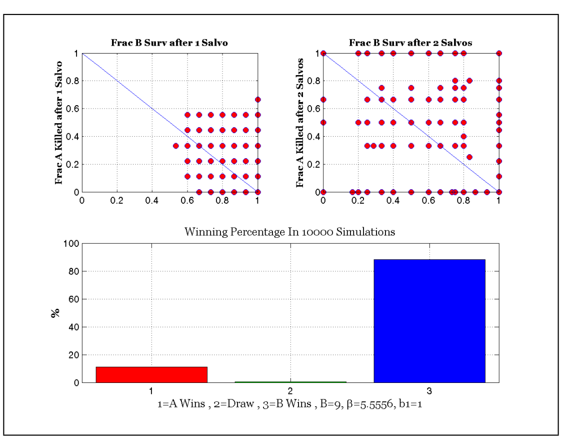

Figure 39. 9 B platforms, with b_1=1, Status After 1 and 2 Salvos

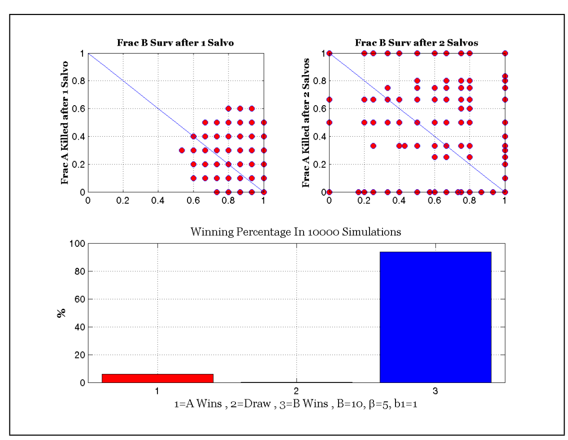

Figure 40. 10 B platforms, with b_1=1, Status After 1 and 2 Salvos

7. Conclusions From Stochastic Salvo Analysis

The deterministic and stochastic form of the Salvo Equations both demonstrates the advantage of numbers and distribution over power concentration. In both Stochastic Models, it clearly shows that that not only do numbers play an important role in ensuring the force survives but that the control of information is also critical. An increase in numbers and distribution increase the uncertainty and variance of information. This implies that a distributed force has two favorable effects,

- It dilutes enemy firepower.
- It complicates the information acquisition problem of an enemy for targeting purposes, and exploits poor information (collected by the enemy) to achieve a higher force effectiveness.

Both Stochastic Models also warn strongly against concentration of power. All results indicate that a very powerful single platform is not suitable against asymmetric threats. The most important reason is that the staying power of a ship usually does not increase in the same proportions as its offensive and defensive powers. This implies that for a fixed number of leakers, the degradation of the force effectiveness of a concentrated fleet is very much greater than that of a distributed fleet.

8. Discussion of Stochastic Salvo Results vs. Deterministic Salvo Results

We now look at the results generated from the deterministic Salvo Model and compare it with the Stochastic Models. Let us take the case when B is distributed over 10 ships, and A has 15 ships, each capable of launching 4 missiles in a single salvo. B has a total of 40 defensive missiles to combat this threat. From the deterministic point of view[11], the fractional loss of B's forces, as given by the Salvo Equations is calculated as follows:

[11]The parameters have been aggregated for the deterministic case

$$\frac{\Delta B}{B} = \frac{60 - 40}{10 \times 1}$$
$$\Rightarrow \frac{\Delta B}{B} = 2$$

The deterministic form shows that each salvo exchange has resulted in a 100% overkill of B! How is it then the stochastic model shows otherwise? We must re-examine the underlying assumptions of the deterministic Salvo Model. It assumes that A has perfect knowledge of B's ships and that A has a perfect distribution of missiles. This implies that any overkill that A achieves with one platform of B is transferred to the next platform of B for a cumulative effect. For example, if A launches 10 missiles at a single B platform, B is able to shoot down 4 of those incoming missiles. The remaining 6 incoming missiles still are heading in his direction and it takes only 1 of the 6 incoming missiles to put that particular B platform out of action. In the deterministic and aggregated form of the equation, the remaining 5 missiles are **redirected** to other platforms of B's fleet. This assumption (for the deterministic case) is debatable because it is, at this particular moment of writing, virtually impossible for a commander to decide if a ship can be considered to be "put out of action". In chapter 6 of *Fleet Tactics and Coastal Combat* [Ref. 9], CAPT. Wayne Hughes states,

> *"...there is a propensity for the attacker to pour shots into a visibly crippled ship until it is seen to sink, even at the cost of letting other enemy ships continue to fight."*
>
> — CAPT. Wayne Hughes

This "propensity" causes a force to expend more resources than what is necessary to achieve a firepower kill. In comparison, when the Stochastic Model is used, these 5 missiles overkill will not be redirected and will merely be registered as **wasted missiles** because there is no carry over effect. This aspect of missile wastage due to imperfect targeting is a real war phenomenon that is not well expressed in a simple and deterministic model.

A point worth noting in this discussion is that it highlights the importance of Battle Damage Assessment (BDA). This example shows that with more efficient BDA systems, one can achieve a better allocation of military resources. This aspect is discussed in a later chapter.

F. MINI-STUDIES

Between the months of August 2001 and September 2001, students of a Campaign Analysis class taught by CAPT. Wayne Hughes, were tasked to analyze the capability and suitability of a distributed force for various scenarios. Basic wargaming was conducted on these scenarios to investigate the potential of a distributed force. A short summary of each scenario is listed.

- **A Peer Competitor Scenario**: A hypothetical peer competitor in the East invades the contested Spratly Islands and intends to extend her sphere of influence down to South East Asia. Her Navy consists mainly of coastal crafts, destroyers, land-based fixed wing aircraft, and diesel submarines. The U.S. decides to intervene in support of her allies in South East Asia. In the first phase, a Crossbow task force[12] based in Singapore, is immediately launched to prevent enemy forces from further build-up on the Spratlys. In the second phase, a similar task force, augmented by two conventional Carrier Battle Groups is tasked to repel the enemy from the Spratlys.
- **A Theatre War Scenario**: Two U.S. Allies in the Mediterranean are poised for a major showdown over islands in the Aegean. The U.S. is committed to deter conflict with a naval presence in the region. Should deterrence fail, the U.S. is poised to strike against the aggressor nation. U.S. Forces include AEGIS ships, and a Carrier Battle Group. The operation is conducted in a littoral background. U.S. Forces are prepared to use Crossbow task forces to combat the aggressor along her own coasts. Aggressor assets are mainly land-based attack aircraft, destroyers with missiles, missile ships, patrol crafts and submarines.
- **An Anti-Piracy Scenario**: The Crossbow task force is used to conduct anti-piracy operations in the Malacca Straits.

[12]The task force consists of small aircraft carriers, and many small, fast, and lethal surface combatants.

- <u>**Maritime Support Of Allies in the Mediterranean**</u>: A U.S. ally in the Middle East is threatened by a coalition of Middle Eastern States. Another ally in the Mediterranean assists the beleaguered nation. However, supplies are running thin and the U.S. has committed a Crossbow task force to preserve the lifeline of both allies.

1. Results From Mini-Studies

The results obtained from the analysis offers a variety of lessons. We will avoid discussions on specific numbers, as most of the results are scenario specific. Instead, we will discuss general results and trends that are of relevance to this thesis.

- <u>**Numbers Can Buy Staying Power**</u>: This conclusion reinforces our initial discussion on using numbers to "purchase" more staying power. One scenario showed that an increase in the number of ships is equivalent to increasing the per unit staying power (of the fleet) by 12%[13].

- <u>**Littoral Warfare**</u>: In littoral warfare, where the enemy is usually characterized by having large numbers of missile boats, a distributed task force will outperform a task force which has most of its assets concentrated in a few platforms. The Measure of Performance used in this case is the fraction of forces surviving after missile exchanges.

- <u>**A Complementary Strategy**</u>: When distributed forces are used as a complement to the "un-balanced" or concentrated fleet, the overall survival fraction significantly improves. This can be shown mathematically with the Salvo Equations and the models that we have used. With reference to our earlier discussions, the reason this occurs is that when both forces are used together, the distributed task force "dilutes" the enemy's fire and draws away the missiles, which would otherwise have been targeted only at the high value targets. A point worth noting is that there is an implicit assumption that the enemy's aim is to inflict damage on ALL ships in the fleet[14].

- <u>**Area of Coverage and Response Time**</u>: In operations which cover a large expanse of area, a distributed force is preferable. This finds particular relevance in anti-piracy operations. It is important to note that for a distributed

[13]Documented in a Campaign Analysis report–*A Peer Competitor* by Rick Muldoon, Lawrence Lim, Cory Culver and Gerry Raia. This report may be referenced in the final report on the Crossbow Project 2001.

[14]This is a reasonable assumption because when the fleet is distributed, each small combatant is still able to inflict significant damage on the enemy. Therefore the enemy must attempt to put ALL ships out of action, because if he allows one or two ships to passage unhindered, he is likely to sustain significant damage.

force to be effective not only must it be able to disperse; it must also be able to concentrate on demand. Therefore we conclude that an effective distributed force is one that is able to disperse and concentrate on demand.

- **Mine Warfare**: The probability of an enemy using mine warfare to enhance its littoral defense poses a great threat to any force, distributed or not. At the moment of writing, a study [Ref. 14] is being conducted on the possible concepts of mine countermeasures applicable to a distributed force.

- **Logistical Limitations**: A ship constructed for use in a distributed fleet must be small and expendable. This limits its endurance and range. Therefore, a distributed fleet must be complemented by an appropriate logistical concept in order for it to be effective. The next chapter discusses this issue.

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V. LOGISTICS ESTIMATES FOR A DISTRIBUTED FORCE

A. INTRODUCTION

We realize that distribution of a force does not come without any costs. One of the main drawbacks of having a distributed force is the refueling of the distributed force. This chapter assumes a notional distributed fleet in the form of a Crossbow task force and estimates the number of support ships required and the size of the support ships. This chapter is taken from a Campaign Analysis paper written by the author in September 2001.

B. BACKGROUND

The Crossbow task force is assumed to be composed of 20 small surface combatants, dubbed **Sea Lance**, supported by 8 small UAV carriers, termed **Sea Archers**, each carrying 8-10 unmanned aircrafts. Support ships, known as **Sea Quivers**, augment the task force.

C. ASSUMPTIONS

The assumptions for the calculations are as follows:

- The "Unrep" process is modeled on the premise that the main component of "unrep" time is the time taken to re-fuel a ship;
- The **Sea Quiver** can only resupply one ship at a time;
- The **Sea Quiver** is the only ship in the task force capable of providing fuel resupply.

1. Definition of Parameters

The parameters are defined as follows:

- k_1 = The fuel consumption per ship in number of gallons per hour;

- k_2 = The re-fueling rate in gallons per hour;
- t_1 = Time at which the first ship is refueled;
- t_m = Time taken in between supplying ships[1].

2. Description of Analysis

At time t_1, the amount of fuel consumed by the first ship is $k_1 \times t_1$. Assume that the standard procedure is to refuel the ship to full capacity, i.e., the amount of fuel that is transferred to the first ship is $k_1 \times t_1$.Therefore, the time taken to refuel the first ship is $\frac{k_1 \times t_1}{k_2}$

Now, the second ship has been sailing for

$$t_1 + \frac{k_1 \times t_1}{k_2}$$

But we must include the time taken for the **Sea Quiver** to move to the second ship, t_m. Therefore, the total time that the second ship has been sailing without refueling (denoted by t_2) is

$$t_2 = t_1 + \frac{k_1 \times t_1}{k_2} + t_m$$

The corresponding amount of fuel that has been consumed by this second ship is $k_1 \times t_2$. Therefore the corresponding time taken to refuel this ship is $\frac{k_1 \times t_2}{k_2}$. Now, by the same reasoning, the time that has elapsed before the third ship starts to get refueled is t_3, where,

$$t_3 = t_2 + \frac{k_1 \times t_2}{k_2} + t_m$$

In general, the time that elapses before the n^{th} ship gets refueled is

$$t_n = t_{n-1} + \frac{k_1 \times t_{n-1}}{k_2} + t_m \tag{V.1}$$

[1]This time will include the time taken to rig the ships together for refueling and the time taken for the two ships to sail to a position in which refueling can occur.

D. ANALYSIS

Assume that the parameters are assigned values shown in Table XI[2].

k_1	1025 gal/hr	The fuel consumption per ship in number of gallons per hour
k_2	24,000 gal/hr	The re-fueling rate in gallons per hour
t_1	24 hr	Time at which the first ship is refueled
t_m	1 hr	Time taken for the supply ship to transit from one ship to another

Table XI. Assigned Values for Model

1. Results Obtained From Model

The graph of the Equation V.1 with inputs from Table XI is plotted and may be found in Figure 41. An zoomed-in graph, Figure 42 is provided for more details:

2. Discussion of Results

From Figure 42, it can be seen that if the maximum time that a single ship is allowed to steam without refueling is 48 hours, then the total number of ships that a single **Sea Quiver** can support is about 11. This is seen by drawing a horizontal line at 48 hours, and checking the point at which this horizontal line intersects the curve. Therefore the number of ships supportable by a single **Sea Quiver** is only about 11.

If we assume that there are about 30 ships in the Crossbow Task Force, then the Task Force would require at least 3 **Sea Quivers**. However, this means that the **Sea Quiver** is working all the time. To ensure adequate operational recovery time and increased robustness through redundancy, we conclude that a single Crossbow Task Force will require at least 4 **Sea Quivers**.

[2]These numbers were obtained from discussions with TSSE (Total Ship Systems Engineering) students. They represent typical fuel consumption rates of small ships. Therefore the calculations here are **conservative**.

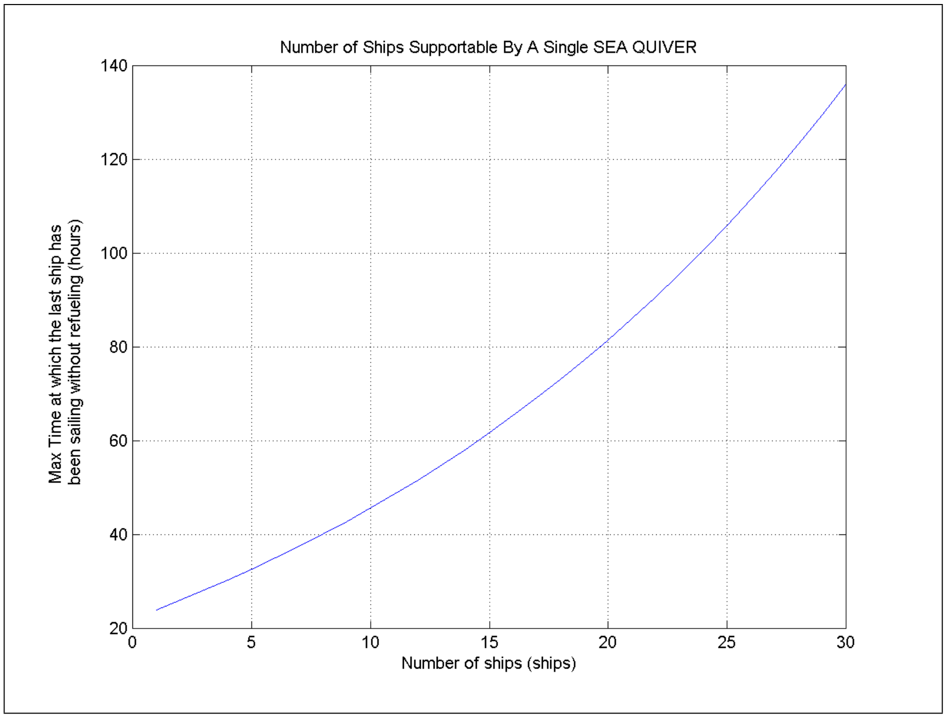

Figure 41. Number of Ships Supported by a Single **Sea Quiver**

E. SUPPORT SHIP SIZE ESTIMATION

This section attempts to estimate the size of a single **Sea Quiver** based on earlier calculations that we have 4 *times* **Sea Quivers** in the task force. We will still use the assigned values in Table XI.

1. Estimation Assumptions

- Assume that any ship in the Crossbow Task Force must be capable of operating 72 hours (3 days), without any refuel from the **Sea Quiver**.
- Assume that all the ships have the same rate of fuel consumption.
- Assume that a single Crossbow Task Force is capable of independent operations for a maximum duration of 168 hours (7 days).
- Assume that the Crossbow Task Force is composed of 30 surface combatants, excluding **Sea Quivers**.

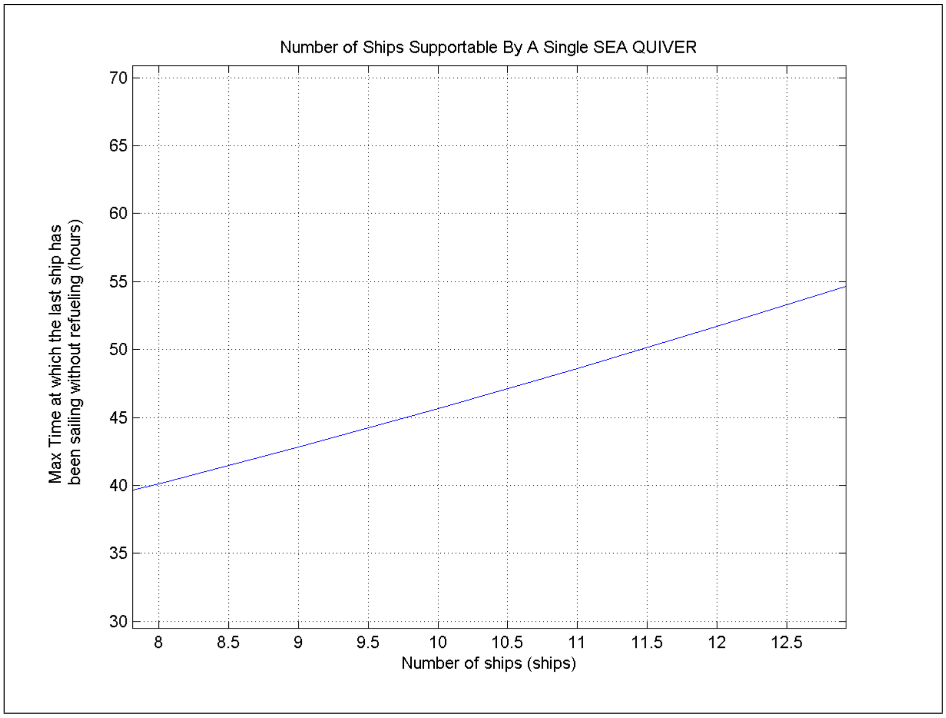

Figure 42. Number of Ships Supported by a Single **Sea Quiver** (Zoomed-in)

2. Total Fuel Required

The total amount of fuel that would be required by the task force for the entire 7 days is given by (Rate of Fuel consumption per ship in gallons per hours) x (Total Number of ships) x (Total Mission Time in hours)

$$1025 \times 34 \times 168 = 5,854,800 \text{ gallons}$$

3. Amount of Fuel Stored in Logistic Ships

But out of this amount, $\frac{4}{7}$ of this is stored in the **Sea Quivers** (since ships can operate for 3 days without refueling from the **Sea Quivers**, out of a period of 7 days).

$$1025 \times 34 \times 168 \times \frac{4}{7} = 3,345,600 \text{ gallons}$$

But since there are 4 SEA QUIVERS, each will carry

$$\frac{3,345,600}{4} = 836,400 \text{ gallons}$$

Based on a simple comparison to AOE tankers. A single AOE tanker carries 1.9 million gallons of fuel. It displaces about 48,000 tons. Since the **Sea Quiver** carries about 900,00 gallons of fuel, (about half that of the tanker), it can be estimated that most probably the **Sea Quiver** will be about 24,000 tons (half that of the tanker). Linear scaling is assumed here, as the quantities being discussed here are volumes.

F. RESULTS, DISCUSSION, AND CONCLUSIONS

If the minimal number of support ships is used, they will turn out to be relatively large ships. Since the distributed task force is unable to carry out its mission without the support ships, it has now become a high value target and will be a "center of gravity" of the task force. This will affect the robustness of the task force[3].

The size of the support ship will make it extremely difficult to keep up with the envisioned speed of the surface combatants. In selecting an appropriate concept for the support ship, it is important that the defense or survivability of the support ship will not consume a disproportionate amount of combat resources.

[3] We define a robust task force as one that will be able to continue to operate even with the loss of a reasonable number of support ships

VI. QUALITATIVE ANALYSIS

A. INTRODUCTION

This chapter will present qualitative discussions pertaining to distributed forces. We will also discuss some real world analogies of distributed forces.

B. ORGANIZATION

This chapter is subdivided into the following discussions:

- Distribution and Soft Kills;
- Concentration of Firepower;
- Torpedo Attacks;
- Rescue Operations;
- Lessons from the Yom Kippur War;
- Lessons from Land Warfare;
- Guerrilla Warfare and Terrorist Networks;
- Information Collection;
- Missions Orientated to Small Forces;
- Lessons from "The Pebble Analysis";
- Distributed Forces are not Designed for Independent Operations[1].

The first section will discuss the effects of ship size and soft kills. The following section discusses the vulnerability of a concentrated fleet when faced with an enemy who is well versed in the principle of concentration of firepower. The third section is an oddball in this chapter because it invokes simple mathematical probability concepts to discuss the effects of distribution with respect to torpedo attacks. The fourth

[1]The term independent means the use of an individual unit of the distributed force to conduct individual operations.

section discusses how distribution will impact rescue operations in the event of a ship being put out of action. The fifth section discusses two naval engagements (involving the use of distributed forces) from the Yom Kippur War. The engagement illustrates how a small, distributed fleet can be employed against an enemy in the littorals—even when out-ranged by enemy missiles—and the concept of operations for a distributed force. It also highlights certain necessary characteristics that a distributed force requires. The sixth section discusses some analogies between tactics employed in land warfare and naval warfare. The seventh section discusses examples of organizations that exploit distribution to achieve a high degree of force effectiveness. The question about the effectiveness of distributed information collection efforts will be discussed in the seventh section. Section eight discusses the nature of certain naval missions that are more suitable for small distributed naval forces. The last section discusses the reasons why a distributed force is preferable, if one of the U.S. Navy's main missions continues to be one of deterrence, and of maintaining a presence around the world.

C. SOFT KILLS

The concept of using soft kills as a defensive measure is an important part of a ship's defense. Soft kills are based on *threat avoidance* as opposed to hard kills, which espouse *threat destruction*. Soft kills would include countermeasures such as seduction chaff, decoys, evasion, and distraction methods.

1. Ship Size and Effectiveness of Soft Kills

Seduction chaff acts in a manner to distribute a cloud of aluminium strips to "seduce" an incoming missile into believing that the ship is actually that cloud of aluminium strips, directing the missile away from the ship itself. (Provided the chaff has been fired early enough, and that it was in the right direction).

However, as the ship size increases, the effectiveness of chaff decreases as it is more difficult to "seduce" an incoming warhead into believing that a cloud of chaff is actually a 90,000 ton ship, as it is more difficult to fake a signature of such magnitude[2]. The larger the warship, the more attractive it becomes to employ hard kill methods for its self defense.

Though hard kill methods are a "safer option" in that the threat is physically removed, there are limitations to hard kill methods. The greatest limitation to a ship that depends primarily on hard kills for its self defense, is its vulnerability to a large, concentrated salvo of missiles.

2. Soft Kills vs. Hard Kills

From our earlier discussion on the Network Model, it is obvious that if the fire control radars are all being used, the ship will be very vulnerable to any additional threat which surfaces during this period of time. In a situation where a concentrated, simultaneous attack occurs, soft kill becomes an attractive option. Let us discuss a simple example.

Assume a ship is able to track a maximum of three incoming missiles at any one time. The ship will sustain a hit if there are four missiles simultaneously homing in on it. But chaff cloud that deflects one missile with a high probability will also deflect all other missiles that arrive at the same time. Schulte [Ref. 19] concluded, using historical records of actual ASCM attacks on warships, that when the ships mounted a successful defense, in every instance soft kill was used. Although in some instances hard kill may have been attempted too, there was no certain instance recorded in unclassified literature when a hard kill was the cause of successful defense. From an economy of effort view, soft kills are more efficient and more economic than hard kills. As the threat of saturation attacks increases, soft kill mechanisms will play a more instrumental role in the defense of a ship. Woe betides the commander whose ship is not defendable by soft kills.

[2]Such a countermeasure may be technologically feasible, but the size of the chaff cloud required is large. Consequently, the volume that is required to store the chaff is also large, making it unattractive (in terms of volume space on board a ship) as compared to a hard kill mechanism.

In situations where the stock of defensive missiles is exhausted, soft kills become the only form of self defense of a ship. In such situations, larger ships are at a great disadvantage compared to smaller combatants. Furthermore, since it is safer and more practical to use hard kills for large ships than small ones, large ships may not be designed to rely on soft kill defense.

3. Evasion

Large ships are inherently less mobile than small ships. This is not only due to their large mass, but the amount of area that they present as a target also decreases the probability of using evasion as an effective soft kill. This is best illustrated by an example. Refer to Figure 43 for details.

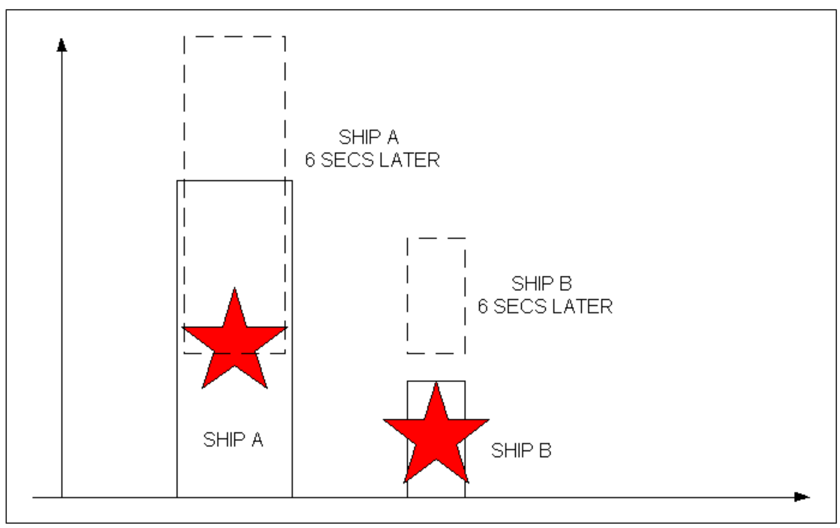

Figure 43. Evasion is not a Function of Speed Alone

Assume that both ships have the same speed. Assume that the enemy is a bomber aircraft. He carries a single 1000-lb dumb bomb. Let us assume that he successfully aims for the centers of both ships and the bombs will hit the ships in about 6 seconds. Assuming that a ship cannot move in a direction perpendicular to its length, after 6 seconds, the small ship has moved completely out of its former location to a new position (shown by the dotted line), however, the larger combatant, even though moving at the same speed, is still going to get hit by the bomb. The same argument holds for line-of-sight weapons without homing mechanisms[3]

A seeker missile has a certain *sweep width*, over which it searches for its target. The sweep width of a missile is equivalent to the *field of fire* of an Infantryman. It does not take much imagination to conclude that a larger object within a field of fire is much easier to hit, as compared to a target that is small compared to the field of fire. The same principle applies to the seeker missile. Also, the sweep width is a function of radar cross section. An ASCM is more likely not to acquire and home in on a ship with a small radar signature, this is especially true when stealth properties are designed into a ship. A seeker missile that has a constant *sweep width*, will home on a larger target more easily than on a smaller one. A small and fast ship will take a shorter time to remove itself completely from the field of fire as compared to a larger ship, forcing the missile to attempt to re-acquire it. The time that elapses during the re-acquisition process could easily make the difference between life and death of the ship. Therefore, the larger the ship is, the less likely it is to employ evasion methods as part of its soft kill defense suite.

Consider the case where a fleet has its power offensive concentrated on a 90,000 ton ship. Its defenses consist of smaller ships that patrol around it. In many cases, ASCMs will home on the target with the largest radar cross section. The implication is that the bigger the ship in the formation, the more likely it is to be targeted by an ASCM.

4. Footprint of a Ship—A Bomber's Confusion

Let us assume that the pilot knows he has to lead with his bombs in order to hit the ships. He calculates the possible positions of the both ships after a time

[3]The 3 minute rule says that a ship moves 100 yards multiplied by its speed in knots every 3 minutes. A 30 knot ship will move 3000 yards in 3 minutes, or 500 yards in 30 seconds, or a 100 yards in 6 seconds. A big ship is approximately 300 yards long. If aim is at midpoint the ship would hit 50 yards from the stern if the time of flight is 6 seconds. A small ship is approximately 50 yards long. The bomb would fall 75 yards astern.

corresponding to the time of flight of the bomb. The big ship's possible positions after one minute will involve many overlapping areas because the footprint of a large ship is large. The pilot will concentrate his bombs in these regions to achieve the maximum probability of hitting the ship. In a similar time duration, the smaller ship will have many more distinct and separated positions with minimal overlap of footprints as compared to the big ship. Therefore, the pilot will have to drop his bombs over a larger area to achieve the same probability of hitting the small ship. If the number of bombs are limited, the probability of hitting the big ship is much higher than hitting the small one. In general, small ships are more difficult to target with bombs because of their small footprint and because their positions are more difficult to predict.

5. Lessons Learned

This section illustrates three issues. Firstly, both size and speed must be considered to obtain a measure of manoeuvrability. Secondly, the smaller the ship is, the less predictable and the more difficult it is to target it. Therefore it is not unreasonable to state that the larger a ship is, the more difficult it is to defend using soft kills techniques of evasion, EW or confusion. Lastly, small distributed forces are more adept in employing soft kill defenses because of their size.

D. CONCENTRATION OF FIREPOWER

The term *concentration of firepower* complements the strategy of *divide and conquer*. In his book [Ref. 9], CAPT. Wayne Hughes identifies one of the great constants of all naval battles as *firepower*.

> *"... It is all the more important now for a tactical commander to have the means to concentrate effective firepower and deliver enough to accomplish his mission before the enemy can bring decisive firepower to bear..."*

1. An Invitation for Concentration?

As noted, a large, high value ship will stand out among its smaller consorts and serve as a beacon for attraction of fire. It will be the prize of the fleet that will invite a concentrated effort to put the capital ship out of action so as to severely reduce the force effectiveness of the fleet. Such a ship is susceptible to a concentrated attack or counterattack. Let us illustrate this point with another analogy.

A scuba diver is armed with a spear gun. He finds himself face to fact with a high value, heavily armed threat, in the form of a shark. His chance of survival is 50-50, depending on who attacks effectively first. Now put him against a school of piranhas. For him to survive, he must be able to target EACH piranha to effectively launch a pre-emptive attack or mount a successful defense. He does this with difficulty as he has a limited field of view (limited scouting capability) and a limited number of spears, (limited number of defensive capabilities) worsened by the fact that each spear is an overkill for each piranha.

The number of piranhas being spread over a distributed volume allows them to attack from any direction. This implies that the diver has to protect himself in many more directions, not allowing him to concentrate his efforts in a single defensive thrust. Because of the number of piranhas and their diminutive size he also has a greater difficulty trying to locate them.

In applying the analogy to naval combat, two insights can be extracted: First, a distributed force attacking from many different directions upon a single objective will not allow the enemy to fend off the attack with a single defensive manoeuvre. Second, the dispersion of units in a distributed force coerces the enemy to spread his surveillance assets over a larger area.

2. Massing or Dispersal?

> *"Whether to concentrate or divide your troops, must be decided by circumstances."*
>
> — Sun Tzu

There will be situations in which a fleet has to mass its assets together for operational purposes. This will be done in situations where massing offers an opportunity to strike a first decisive blow to the enemy. Or, when there are joint defenses protecting the fleet, such as a CAP (Combat Air Patrol), massing offers a better defensive umbrella. However, if there are no advantages to be gained from massing (apart from economic reasons and convenience issues), then dispersal is better option. Imagine a fleet of ships trying to sneak in to surprise its enemy, in such instances, massing often does offer a higher probability success. However, if the ambush has not been delivered successfully, a speedy dispersal before the counterattack is required.

From the defensive point of view, we draw a parallel analogy to an Infantry battalion in defense. An integrated defense of an objective by massing three companies together is more effective than if a single company defended it. The main reason is because when an area is divided into different sectors, each company is able to focus its effort and firepower on its own sector. When there is only one company, its sector of responsibility is much larger. Consequently, its firepower and surveillance assets must be spread over a larger area, which degrades the effectiveness of the defense.

3. Lessons Learned From Concentration of Firepower

It can be inferred from this discussion that it will be more difficult to counter a distributed threat. Firstly, to counter a concentrated threat, a single defensive manoeuvre may be adequate to fend off the attack. In comparison, to counter a distributed threat, not only are better scouting capabilities required (higher resolution is required), good fire distribution systems and a tighter defense of all sectors are also required.

We have established that there exist two conditions in which massing forces does indeed offer a better outcome. In cases where a fleet has a range advantage and overwhelming offensive firepower capable of destroying the enemy in a single first salvo, then it makes good military sense to mass forces for purposes of attacking decisively first.

The second case is when massing for defense achieves superior, interlocking mutual support and the strength of the combined defense can overcome any weight of attack the enemy can throw at the formation.

E. TORPEDO ATTACKS

The biggest numerical threat to any surface fleet is from anti-ship missiles and the analytical emphasis herein has been on the missile threat using the Naval Salvo Equations. At times a serious threat is also posed by the presence of enemy submarines and torpedo attacks. The analysis and simulation of torpedo attacks depend on many situational parameters e.g. range, spacing, lateral range etc. We will confine ourselves to a very simple probabilistic analysis of a torpedo attack and calculate the expected **percentage of fleet firepower firepower lost**.

We will assume 2 hypothetical fleets. Fleet A is a fleet of 5 ships, with an uneven distribution of firepower. The distribution of Fleet A's firepower is found in Table XII. Fleet B is a fleet of 10 ships with a uniform distribution of firepower. The distribution is found in Table XIII

Ship	Percentage of Fleet Firepower
1	60%
2	10%
3	10%
4	10%
5	10%

Table XII. Parameters for A's Fleet

Ship	Percentage of Fleet Firepower
1	10%
2	10%
3	10%
4	10%
5	10%
6	10%
7	10%
8	10%
9	10%
10	10%

Table XIII. Parameters for B's Fleet

1. Assumptions

We will assume the following:

- The only cases of interest are those when a torpedo is fired and hits a ship;

- A submarine is only detected when it fires a torpedo;
- Submarines have the first strike advantage;
- A single torpedo hit is sufficient to put a ship out a action.
- All ships in a fleet have the same probability of being targeted by the submarines. This is a **conservative** assumption, because a submarine can select a preferred target–usually the highest-valued ship.

This simple calculation does not consider the defensive measures that a ship can take against a torpedo. We are interested in a relative comparison so we will postulate that any defensive measures employed will affect the survivability of both fleets equally. We will only focus our attention on the effects of distribution, *ceteris paribus*[4].

2. Expected Percentage of Fleet Firepower Lost per Enemy Torpedo Salvo

Using the expression for expected values,

$$E(x) = \Sigma x p(x)$$

Fleet A has 5 ships, therefore the probability of putting each ship out of action is 0.2. Calculating the expected value of the losses accrued from a single enemy torpedo salvo for Fleet A, we find,

$$= (60)(0.2) + (10)(0.2) + (10)(0.2) + (10)(0.2) + (10)(0.2)$$
$$= 20$$

Fleet A will be expected to lose 20% of his total fleet firepower with each enemy torpedo salvo. However, for Fleet B,

[4]From modern Latin, literally "other things being equal."

$$= (10)(0.1) \times 10$$
$$= 10$$

Fleet B will only be expected to lose 10% of his total fleet firepower from the attack.

3. Discussion of Results

The results show that the number of ships in a fleet, and the distribution of firepower among a fleet will contribute to a lower percentage of fleet firepower lost per enemy salvo. This occurs because when a fleet has more ships, each ship will have a lower probability of being the target of a torpedo attack, assuming that the enemy distributes his torpedoes equally among the ships of a fleet. The situation is worsened for Fleet A if the enemy concentrates his torpedo attacks on the ship that carries the most amount of firepower[5].

After the submarine(s) have launched their torpedoes, their positions are compromised they are subject to counterattack and they will attempt to evade the ASW assets. This causes a dilemma among the enemy submarines: he will be forced to consider if it is worth it to actually deploy submarines against a distributed fleet since each submarine can only put 10% of the fleet firepower out of action before the submarine turns into the "hunted". In contrast, the enemy will find it very profitable to exchange one submarine if that single submarine has a chance to put 60% of the fleet firepower out of action when it torpedoes the highly valued ship.

F. RESCUE OPERATIONS

So far, we have concentrated on the percentage of fleet firepower lost when a ship is put out of action. It is important to remember that when a ship is put out of

[5]In most cases, high value ships would also carry other important assets of the fleet, e.g. the CIC "nerve center" of the fleet, airborne surveillance assets, fleet logistic supplies, fleet communication assets etc. This compounds the losses further because when that ship is put of action, the entire fleet will be deprived of these assets too.

action, the other ships of the fleet have to assist the disabled ship. Assistance may include protecting the disabled ship from further damage and rescuing personnel from the disabled ship.

If the disabled ship is a high value ship, it is likely that the fleet would be committed to protect the damaged high value ship, to prevent it from further damage as well as to be ready to evacuate personnel. As mentioned earlier, it is likely that the high value ship will be large and carry many more assets than its consorts, including personnel. This would mean that in the event of a high value ship being put out of action, the rescue, protection and evacuation of the personnel and ship would most likely involve a significant portion of the entire fleet, further degrading the offensive capabilities and movement of the fleet. The situation is further aggravated if sailors have to be physically evacuated on board another ship for protection and medical aid. In this case, it is almost certain that all other ships would have take in the extra personnel. The process of rescue and search would almost certainly disrupt the operational tempo of the mission.

From another point of view, if a smaller consort has been put out of action, the high value ship is almost obliged to provide protection and cover for the smaller ship. This is due to the fact that most of the offensive firepower and combat capabilities, e.g. airborne surveillance, central command and control system cannot be adequately provided by the rest of the surface fleet. Again, this effectively ties up the operational capabilities of the fleet.

Now consider if the ships were more equally distributed, with each ship of roughly the same size and capability. In the event of a ship being put out of action, it is likely that the provision of firepower protection and evacuation of personnel could be performed by a similar ship of the fleet, allowing the remainder of the fleet to continue unhindered. As discussed in previous chapters, the distributed fleet would still be able to pursue its mission because it does not lose a significant amount of firepower when one or two ships are diverted to other purposes (as in this case, where it has been diverted to rescue operations).

G. THE YOM KIPPUR WAR

The Yom Kippur and the Six Day Wars provide valuable insights into the military worth of numbers. Rear Admiral Benyamin Telem, an Officer Commanding in the Navy of the Israeli Defense Forces, lectured on the naval lessons of the Yom Kippur War [Ref. 21]. We will relate some of his lessons to show how numbers do make a difference in battle.

1. Background

In the first example RADM Telem cites events on 6 October 1973, when 5 Israeli patrol boats were sent against a force that was almost similar in size, but were more heavily armed[6]. The Arabs had the range advantage and the firepower advantage, with bigger, and longer range missiles[7]. The result of the conflict was one OSA, two Komars, one K-123 Torpedo Boat, one T-43 Minesweeper (all Arab vessels) sunk. The second engagement on 7 October, saw six patrol boats defeat 3 Egyptian OSA boats. In both engagements none of the Israeli patrol boats were put out of action. The Israelis' strategy was to detect the enemy from far, charge in, evade all enemy missiles through a mix of EW and skillful manoeuvres, before closing in with their own missiles and further closing in to finish off the enemy with naval gunfire.

2. Heavily Armed Ships Are Not Invulnerable

The first lesson to be drawn is that large[8] ships, with long range weapons and great standoff capability are not invulnerable to attacks. It may be argued that these ships should have been augmented with a strong defensive capability to complement their offensive capabilities. However, a ship in which self defense become a top priority

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[6]Consisting of Egyptian and Syrian naval Forces

[7]Mainly Russian Styxs

[8]In this engagement, the ships on both sides were small, both sides no more than 500 tons, but the Egyptian Komar ships were armed with large–needlessly large–missiles

will inevitably not only be a liability to itself in battle as it would be more concerned about defending itself that it would focus on defensive tactics, it would also consume scarce resources which could have otherwise been committed to the offense.

> *.....boats,* **under no circumstances** *become big or expensive in equipment to the extent that their own defense becomes a first priority requirement in itself. This would inevitably negate their offensive capability [Ref. 21].*

—RADM Benyamin Telem

3. Risk, Victory, and "Combat Consumables"

The second important lesson to be drawn is that risk is an inevitable factor in every battle. The Israeli's move to charge in and close the distance to engage the littoral forces was a very risky, but necessary, move. In hindsight, it was a well taken risk. The Israelis were inclined to risk the ships because they knew they still had the potential to win the battle even if they lost a part of their fleet. They had five boats to gamble on, each boat was armed with *Gabriel* missiles that were designed to put a larger[9] enemy ship out of action with one hit. Each boat was 250 tons, with *Gabriels*, 40mm and 76mm guns, all fully radar controlled. The long term strategy was to produce this affordable boat in numbers.

The loss of a single Israeli ship would not have jeopardized the entire mission—the ships were "combat-consumable". The Israelis could afford to lose one or two of those ships, and still win. This fact emboldened the Israeli Commanders to gamble and take the risk to achieve victory. However, the same cannot be said of modern fleets structured around few platforms which possess most of the fleet's fire power.

There are numerous examples in military history, in which, the side willing to take the risk has triumphed decisively over its adversary. There is no better example than Generalfeldmarschall Erwin Rommel in the great North Africa Campaign. He was a constant risk taker who saw the need to exploit opportunity at the critical time,

[9] Larger than the Israeli ship that was carrying the missiles.

and used it to the Allies' misfortune. The presence of units with strong offensive powers, but which are "combat consumable[10] gives a commander the latitude to take risks to win decisively. To enact any military decision, to some extent, is a risk. A fleet with high value vessels is generally likely to be more risk averse, and consequently, is less likely (as compared to a force equipped to take calculated risks) to achieve a great victory.

4. Combine and Permutate to Achieve Surprise

Surprise is one of the principles of war, requiring unexpected approaches in normal situations. A simple mathematical argument is followed here. Commander A with 10 similar ships, wishes to analyze the task forces that he can assemble. He may choose to assemble 2 task forces, and he has many ways to choose them. He may select a 9-1 combination, an 8-2 combination, and the list goes on. He may even elect to have 3 task forces. After appreciating the situation, Commander A has a range of flexible options that he may choose to configure his fleet. The enemy will find it difficult to plan an operation against all combinations.

Commander B has 10 ships as well, but his 10 ships are all different. One of the ten ships is a high value ship. It has weak defenses, depends on the other ships in the fleet for protection but carries almost 90% of the fleet's total offensive capability. Commander B has fewer options for configuring his fleet; he is almost obliged to concentrate his forces around the high value ship. An enemy who is trying to figure out the possible fleet configurations for Commander B will have a much easier problem, as compared to the former. Commander B's fleet will eventually present much less of a surprise to the enemy than did Commander A's.

[10] "Combat consumable" in this context, means that victory that still be achieved even if part of a fleet is put out of action."

5. Diversify to Reduce Vulnerability

The last lesson from the Yom Kippur War is minimizing the possibility of catastrophic losses, akin to what many people call putting "all one's eggs into one basket". We will again quote from RADM Benyamin Telem [Ref. 21].

> *"No doubt numbers of boats will count in any future engagement, enabling dispersal of forces as well as minimization of losses through direct hits"*
>
> —RADM Benyamin Telem

6. Soft Kills and Electronic Warfare

Though RADM Telem does not divulge the secret of the Israeli Navy's success in the evading the Egyptians' Styx missiles, to get within reach of the *Gabriels* missiles, other sources can confirm that it was through a mix of soft kill techniques, coupled with skilful manoeuvring that the Israelis managed to avoid the *Styx* missiles.

7. Summary of Yom Kippur Lessons

The summary of lessons learned from Yom Kippur war are as follows:

- High value ships with long range missiles are not invulnerable to attack. Small packets of suitably armed patrol boats can defeat them. Therefore, avoid massing power onto a few platforms, for they are attractive targets for concentrated attacks.
- Risk is an integral part of battle. Without risk, great opportunities cannot be exploited. The ability to win with just a part of the fleet gives a commander the latitude to take calculated risks. If high value ships are the only ships that can win a battle, it makes a commander risk averse, limiting his options of exploitation of successes. The advantage of having a large number of small, inexpensive ships is clearly illustrated in this example.
- A fleet of small ships offer more combinations and permutations, which can be easily configured to suit the situation. A fleet with few ships have fewer options for combination and permutations, the interdependence of these ships further reduce the number of possible configurations. The fleet with a few ships will present less of surprise to an enemy than a fleet with many ships.

- Numbers reduce the possibility of catastrophic losses and minimize overall military losses through diversity. The more distributed a force is, the greater the reduction in single points of failure.
- History shows that a distributed force is well suited for the conduct of littoral warfare.
- Electronic Warfare and soft kills when exploited, raises the combat effectiveness of a fleet.

H. LAND WARFARE

> *"If equally matched, we can offer battle; if slightly inferior in numbers, we can avoid the enemy; if unequal in every way, we can flee from him."*
>
> —Sun Tzu

The importance of numbers has always been recognized in land warfare. Practitioners of land warfare have never advocated building a tank with ten turrets, anti missile capability and carry a battalion's worth of firepower. The reason is simple: Land warfare practitioners have recognized that by doing so, they would be opening themselves up to concentrated attacks. The second reason is that Land warfare practitioners have recognized that high value assets cannot be everywhere at the same time. Therefore firepower on the land has never been concentrated in the hands of a few. Rather a distributed capability that can be concentrated or dispersed on command have been preferred.

Land warfare tactics, though different from naval tactics, have some principles that can be applied to naval tactics in the littorals. The principles for an Infantry ambush operations require stealthy movement, superior scouting, superior coordination, application of a concentrated barrage of fire on a surprised enemy, and a quick dispersal before surviving forces are able to launch a counterattack or call for artillery fire. When defensive powers are strong, instead of a quick dispersal and withdrawal, a quick concentration may be required to repel counterattacks. Naval tacticians also espouse the values of superior scouting, superior coordination and concentration of firepower, but these tactics are more demanding in a terrain as big as the ocean. The open nature of the seas makes it difficult to apply the two other principles, stealthy movement and dispersal or concentration on demand.

However these two principles are of increased significance in the littorals. Operations in littoral waters are most dangerous in the presence of an adversarial coastal navy. The presence of land clutter, small islands, fishing fleets, and small commercial shipping vessels offer concealed routes of approach for the attacker. Stealthy movement in such operations reduces the possibility of being exposed to a first salvo launched by the enemy. It is difficult to conceal a 10,000 ton cruiser in such circumstances as compared to a smaller ship.

1. High Risk Missions

A final analogy to land warfare is the concept of a reserve. Infantry tacticians have recognized that the manner in which a reserve force is used will often determine the outcome of a battle. For that reason, it has always been practice to place the elite or high value forces as the reserve force. For example, tn the Gulf War, Iraq's reserve was the Republican Guards, the Iraqi elite. They were not used as front-line defenses, rather they were used as strategic reserves. High value assets should never be used to perform penetration manoeuvres; they should be used to exploit the success of a successful penetration. Applying the same lesson to the Navy, a littoral "access" or "penetration mission" is unsuitable for high value assets, because of the large risk involved and because once destroyed or damaged, high value targets cannot be easily replaced. The loss of the high value asset also drastically reduces the combat potential of a military force. They will not be able to contribute beyond the penetration, and hence will not be able to contribute to getting the seat of purpose, which lies beyond the littorals—on land.

I. GUERRILLA WARFARE AND TERRORIST NETWORKS

The perceived principles of guerrilla[11] warfare and terrorists networks are

- Mobility;
- Small Signatures and Stealth;
- Flexibility;
- Persistency;
- Highly Distributed (operation is close to autonomous in some cases);
- Asymmetrical Capabilities [12].

Well founded guerrilla organizations are exceptionally successful in their operations and extremely difficult to combat. They have many attributes that can be applied to a conventional naval force.

1. Mobility

Mobility in guerrilla warfare terms does not refer to speed alone. Rather it means the ability to move in all terrains, in many different directions. A guerrilla unit relies on its ability to *disperse or concentrate* on demand. Guerrillas avoid a pitched battle with the enemy but are advocates of "hit and run" tactics. Such tactics not only constantly wear down the enemy, affecting concentration and morale, but more importantly, forces the enemy to expend ammunition. In the context of modern naval combat, such tactics would be highly successful, because missiles are of a limited supply, and missiles pods and launchers take time to reload. Constant harassment will cause an enemy to deplete his offensive missiles, and thus be open to attack. However, such tactics can only be achieved if the fleet has mobility, stealth and is distributed.

---

[11] We shall mean both terrorists and guerrillas when we use the term guerrilla.

[12] The losses inflicted by the guerrilla or terrorist far exceeds the value of the terrorist or guerrilla. The costs incurred of locating and destroying the guerrilla far exceeds the costs of conducting the guerrilla operation.

2. Small Signatures and Stealth

Guerrilla units rely on their small signatures to avoid enemy detection. They blend easily into the background and make it difficult for the enemy to detect and identify the enemy. Guerrillas, and especially terrorists make it a point that they look like their environment. Applying this lesson to naval combat, smaller ships have better survivability as compared to larger ships because their smaller radar cross sections make them look like innocent commercial crafts.

3. Flexibility

Guerrilla units, and especially terrorist units, do not have fixed and rigid structures. Their composition is fluid and can easily be mixed and matched to meet different purposes. For example, in the Al-Queda network[13], training is focused on developing small and highly distributed units. When circumstances dictate, guerrilla forces can be deployed on demand and concentrated to fight a conventional ground war[14]. Flexibility in structuring a task force offers its practitioners the opportunity to exercise *economy of effort*[15]. When necessary, small regional units could unite for large scale attacks. If enemy pressure became too great, they would break down into smaller units and scatter. In application to naval combat, the construction of high value warships with great striking power reduces the flexibility of a commander. He is unable to configure his fleet to meet the requirements of multiple tasks. He will not be able to exercise economy of effort. He will find it difficult to counter many small scale, geographically separated conflicts simultaneously.

4. Persistency

One of the principles of guerrilla warfare is to constantly harass the enemy to break his morale, his will to fight, and expend his resources. Given the nature

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[13] The Al-Queda is the terrorist network sponsored by Osama Bin Laden

[14] The success of this strategy is yet to be seen

[15] Economy of Effort is one of the Principles of Soviet Operational Art

of modern naval combat, fleets that are designed with a large amount of offensive power concentrated in only a few ships, are only capable of "pulsed" operations. These high value ships are able to operate for a certain timeframe, beyond which they have to conduct resupply operations. If they leave the operating area to do so, almost all of the fleet's firepower is removed, allowing the enemy time to reorganize and recuperate. With a distributed fleet, continuous operations are possible. The distributed fleet might be organized into "shifts" and maintain a 24/7 pressure on the enemy.

5. Locating a Distributed Force

Ironically, the current terrorist crisis is the best example of the effectiveness of a distributed enemy. When an enemy is distributed, there is no focus. There is no single point of focus to concentrate firepower or effort. The effort or cost in trying to locate a geographically distributed enemy is significantly higher than that of trying to locate a single enemy. In modern naval terms, this implies that given fixed scouting resources, it would be much easier to detect and identify a single, large ship, as compared to a distributed fleet. The effort in locating all the individual units of the distributed force is usually much higher than the effort in locating a single force.

Terrorists networks and guerrillas are examples of highly distributed structures. They have also demonstrated, in a most unfortunate manner, the effectiveness of the principles by which they operate. They offer naval planners some useful lessons in using mobility and stealth to induce the enemy to deplete his missile repository. They also demonstrate that with a large number of small value units, there is a great flexibility in mixing and matching a task force, and by distributing its firepower, the fleet is able to conduct continuous operations. And lastly, they have also demonstrated that distribution spreads the enemy's attention over more units, making it more difficult to achieve concentration of firepower and effort. Lastly, a distributed fleet reduces the likelihood of catastrophic mission failure—a possibility with concentrated fleets when only one or two units are put out of action.

J. INFORMATION COLLECTION

There have been many projects focused on the development of independent agents to collect information. The value of information in a conflict is a recognized force multiplier. A study by John McGunnigle [Ref. 16] measures the actual military worth of information in terms of military units[16]. He concluded that information can enhance or degrade a force's effectiveness (depending upon how that information is used), but increasing force advantage always enhances the force's effectiveness. Nevertheless, a combat commander always seeks to obtain information about his adversary to try to use it to his advantage.

1. Distributed Collection vs. Central Collection

It is obvious that a distributed force would be able to be in more places in one time as compared to a force that is highly concentrated. Assuming that both systems use the same surveillance equipment in the form of an airborne asset, the instantaneous area of coverage is much greater for a distributed force. And because the instantaneous area of coverage is larger, the information collected is more accurate as compared to a single source of information.

Consider the example of a monopulse radar. The monopulse radar uses two instantaneous pulses to form a snapshot of a target in time. The difference between the pulses will be used as information to track the target in space. Another moving target radar architecture is to have the radar generate two pulses sequentially and use the two pulses to form a snapshot of the target. However, in the time between the two pulses, the target will have moved, therefore there is a small error involved when using the sequential pulses. A distributed force is able to generate a more accurate picture

[16] In his thesis, he performs three experiments to determine the value of information. One of the experiments included a simple contest involving senior military officers and teaching staff at the Naval Postgraduate School. In some cases, he provided them with extra units and in others he provided them with extra information about the enemy units.

and collect more accurate information at a specific time as compared to a single force. Since a distributed force is able to obtain better information and at a faster rate, and that information can enhance a force's effectiveness, it is not unreasonable to infer that (if information were used in the same manner for both forces), a distributed force would gain more force effectiveness due to the accuracy of its information.

2. Multistatic Radars

In a recent article in a popular science magazine, studies have shown that properties of stealth are greatly diminished when there are many geographically distributed radars[17]. In the distributed fleet, the receivers may be geographically dispersed and therefore would be less susceptible to a surprise or stealth attack as compared to a fleet which was unable to be geographically distributed.

In 1977, K Milne described the advantages of multistatic radars[18] in his paper, *Principles and Concepts of Multistatic Surveillance Radars* [Ref. 17]. His main findings are,

- Moving targets cannot present 0 doppler to all receiving radar sites simultaneously. Hence, the "tangential fades" common in monostatic radars disappears. As a result, simpler doppler processing, with broader zero-doppler gaps, can often be employed.
- The system is virtually immune to deliberate highly-directional interference, since the location of receiving sites, the particular transmissions being used by a receiving antenna's sidelobes are not known by the interferer. By contrast, monostatic radars generally "advertise" their location and sidelobe patterns by transmitting through a common transmitting and receiving antenna.
- Since duplicated or triplicated coverage is provided, any one transmitting or receiving station be shut down and re-located without total loss of information. Similarly, "winking" or "blinking" transmitters can be employed as a counter to physical attacks by missiles which home on to radiated signals.

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[17] In fact, this was the original concept of the radar, known as the bistatic radar. One radar would be used for generating the pulses and another, which was situated in a different location, would be used to pick up the signals and detect any intruders. Stealth works mainly by reflecting the pulses in a direction different from where it originated.

[18] A radar system which uses separated sites for the transmitters and receivers

A distributed platform may be compared to a multistatic radar system, in which some platforms are transmitting and others are only receiving. From Milne's first and second conclusions, it can be said that a distributed fleet is more resilient to jamming and will have a higher probability of detecting stealth-enhanced surface or air platforms. In *Fleet Tactics and Coastal Combat*, CAPT. Wayne Hughes describes a strikingly similar scenario that takes advantage of multistatic radars as such. Milne's last two conclusions also indicate that a distributed system employing multistatic radars will be less vulnerable than monostatic radars.

K. MISSIONS SUITED FOR SMALL FORCES

A navy that does not incorporate force distribution into its fleet must inevitably end up concentrating its power in its large ships. However, there are a variety of missions in which only small forces are required. Such a navy would incur excessive costs by using their large ships to perform missions which do not require such great firepower. The cost arises from:

- Manpower Costs;
- Operational Costs;
- Wear and Tear of Valuable Assets.

The larger the ship is and the more assets concentrated in that ship, the greater the manpower costs and operational costs associated with the operation of the ship. A more significant impact would be the wear and tear on a valuable asset. Large ships are valuable assets but if the navy does not have an array of smaller ships to perform smaller scale missions otherwise imposed on the large ships, the large ships would constantly have to be used. The over utilization of the asset will heighten the possibility of a fatigue failure in the personnel, mechanisms and electronics systems in the ship. Examples of missions in which the use of large ships represent overkill include:

- Deterrence Missions;
- Anti Piracy Missions;
- Anti Terrorist Missions.

With trends of increasing missile lethality, using large, high value assets to perform missions better suited for a smaller force is unacceptably risky and simply wasteful. The introduction of a high value asset in a low intensity conflict exposes the asset to asymmetric attacks. Recent experiences have shown that the ship does not even have to be in direct conflict to expose it to asymmetric attacks. The terrorist attack on the USS COLE in 2001 highlights the vulnerability of ships, anywhere and at anytime.

L. THE PEBBLE ANALYSIS

Graphing An Optimal Grand Strategy [Ref. 1] is a paper by John Arquilla and Hal Fredricksen[19]. The paper studies historical empires and analyzes the span of control of the British and the Roman Empires. Though the United States does not have an empire, as the enforcer of world peace and the only remaining world superpower, it must to be able to maintain military presence overseas. One of the most valuable lessons gleaned from the "Pebbles" study is that it is important to have enough military capability to spread over the "empire".

From the study, we can conclude that an essential requirement for a nation to be able to maintain military presence is that the nation must have sufficient numerical assets to be forward deployed. It is clear that if the military strength of the nation is concentrated in a few, indivisible units—as they have to be deployed as a whole unit—then it is more difficult to maintain a widespread forward presence. The study highlights the importance of military forces being able to concentrate or disperse on demand. From historical data, it shows that the nations that were unable to distribute

[19]He compares military units to pebbles and likens the strategic placement of military forces to the placement of pebbles on a conceptual map of the world

its military forces were unable to maintain their "empire". Similarly if the United States is unable to configure its military to do so, its strategy of having a forward military presence might ultimately fail.

M. DISTRIBUTED FORCES ARE NOT DESIGNED FOR INDEPENDENT OPERATIONS

On the other hand, it is important to avoid the misconception that a distributed fleet would automatically provide more coverage. The distributed force is designed to work co-operatively within a framework in which there are many ships. There exist situations in which individual elements of the distributed force may have the luxury to operate independently and there are situations when this is impossible. The former will occur when the threat is low. However when operating in high threat environments, a single element of a distributed force must not operate in the region as a single unit. It must be deployed as a distributed fleet in such situations.

The reason is simple. A single ship in a distributed fleet is designed to operate as such. Hence its defense systems will be designed around the premise that the single ship is in no danger of being the target of a concentrated attack. Therefore its design will allow it to be less defense orientated and more offense orientated. However, if it is deployed alone and without mutual support, the ship will be the target of a concentrated attack, and will not be able to defend itself sufficiently when that happens.

Therefore, it is important to be able to aggregate the ships of a distributed fleet into a balanced force by having modular weapon systems. When forced to operate independent of the fleet, small task forces must be able to be outfitted with reliable scouting systems, strong defensive weapons and undersea warfare. This is the very essence of the Crossbow concept. When the distributed fleet is split up and sent into low intensity conflicts, it must be configured to detect and attack the enemy while defending itself without the rest of the fleet.

VII. SUMMARY

A. THE CASE FOR DISTRIBUTION

We have examined a modern naval force using three basic models. In all three models there is quantitative proof that there are substantial benefits to be reaped from distributing a force.

First, from a network view, a distributed force:

- uses the C2 resources in a more efficient manner;
- has a more reliable C2 infrastructure;
- enjoys graceful degradation compared to a single entity.

Second, the classic Lanchester Equations for modern warfare, shows that an additional unit added to a force is worth more than increasing the existing firepower of the same number of units. The results indicate that even with no increase in total fleet firepower—total firepower kept constant—a fleet will substantially increase its chances of winning by merely redistributing its combat potential among a larger number of units. In a similar vein, a fleet that reduces its numerical strength in favor of increased combat potential per ship will substantiality decrease the effectiveness[1] of the fleet.

Third, the Naval Salvo Model takes into account the staying and defensive powers of a ship—two important parameters that were not explicitly expressed in the Lanchester Equations. Results from the Salvo Model indicate that an increase in the numerical strength of the fleet is the most effective and efficient method to increase the force effectiveness. This is prominently due to the fact that the staying power of a ship cannot feasibly increase in linear proportions to its displacement or other capabilities. This occurs because staying power is a very difficult quality to design

[1] Measured in the force exchange ratio.

into a ship and the increasing lethality of Anti-Ship Cruise Missiles (ASCM) further compounds the problem.

Results from the Naval Salvo Model shows that a fleet, in which power is concentrated in a few ships, will have a great degree of instability in its force effectiveness. The total firepower of the fleet is significantly reduced if the high value ships are put out of action. As a quantitative measure, the degree of stability of the force effectiveness of a fleet can be characterized by the **percentage of total fleet firepower lost per leaker** (PTFL). In fleets that have a few extremely high value warships, PTFL value is extremely high. In distributed fleets, the PTFL is significantly lower. PTFL is also a measure of the "graceful degradability" or robustness of a fleet. Results from the Naval Salvo Model indicate that for distribution to be attractive, the PTFL for a distributed fleet must exceed the PTFL for a concentrated force.

Two stochastic models based on the Naval Salvo Equations show results that differ from the original, deterministic Salvo Equations. The Salvo Equations in pure form gives both sides perfect distribution of missiles. The stochastic models remove that artificiality by allowing for wasted missiles and imperfect information—both common occurrences in a battle—and each missile engagement is modeled as a stochastic duel. Results from both models show that when total firepower—defensive and offensive—is kept constant, a fleet significantly improves its chances of winning merely through distribution of its combat potential among more units.

In some simulations of the Salvo Models, results indicate that there are specific instances when force concentration is preferred over force distribution. The ability of a distributed fleet to disperse or concentrate on demand makes it extremely adaptable to these instances. In comparison, fleets with rigid formations cannot easily be adapted to the situation.

The number of leakers that a fleet will experience will be a function of its defensive powers. Hard kill missiles aside; soft kill defenses are becoming more important as concentrated attacks appear to be increasingly popular among potential enemies. However, the effectiveness of soft kill defenses is a function of the size of a ship. As a ship grows in size, it becomes less feasible to defend it through soft kill mechanisms.

As a ship grows in size, its military value increases and it becomes an attractive target for potential enemies. Both sides in the conflict recognize this and the attacker will attempt to concentrate maximum firepower on the highest valued ship in the fleet. This causes the defending fleet to devote its resources to the preservation of the high value ships to prevent their loss. The loss of one big ship will significantly cripple the effectiveness of the fleet and may terminate the mission. Distributed forces are designed to prevent this situation from happening. Specifically, distributed forces do not have a center of gravity, or SHOULD be designed specifically without a center of gravity. The description of a well designed distributed force is: a fleet which still maintains significant offensive potential even when it has lost a reasonable number of its ships. The ability to lose a reasonable number of ships and still have the capability to win an encounter gives the commander more confidence in planning **bold** moves to exploit success. The Yom Kippur War is an example of such an instance. During that war, in both land and sea battles, the Israelis demonstrated an extremely high degree of initiative and force effectiveness.

Land tactics are generally very different from naval tactics. However, land combat recognizes that there are many cases in which the mission is too risky to commit high value forces. In such instances, numerical superiority is used to overcome the risk factor. D-Day at Normandy is a good illustration of how the Allies used numerical superiority to overcome a stubborn littoral defense. Similarly, there are very risky naval missions, littoral missions against a coastal navy for example. A navy without a distributed fleet, would be forced into committing its high value assets to perform such high risk missions. Distributed forces provide the numerical superiority to overcome such risks.

B. THE REQUIREMENTS OF A DISTRIBUTED FORCE

Drawing from the Network Model, a distributed force must be supported by a robust and reliable C2 network. The purpose of the C2 network is to link the components of the distributed force together such that they can be dispersed or concentrated on demand. The network has to be distributed too; otherwise it would be the center of gravity for the distributed force and serve to become the high value target.

The Naval Salvo Model also shows that for distribution to be attractive, the sum of the total staying powers of a distributed fleet must exceed the sum of the total staying power for a concentrated fleet. It also implies that in the presence of strong enemy firepower, for a distributed fleet to be effective it must be deployed in totality and should not be split to conduct separate independent operations. However as the situation changes, the distributed fleet can be easily adapted to meet the requirements.

Small ships are characteristic of a distributed force, but small ships have poor range and endurance. The need for a reliable, robust and efficient logistic support system is required to support the force. Similar to the network requirements, such support systems must not be so large that they become the center of gravity of the fleet or else the advantages reaped by distributing the fleet will be temporary and transitory.

It is imperative to note that this study does not argue against the construction of high value ships. Rather it shows that there are advantages to be reaped with distributed forces, and that a balance between a mix of high value assets and distributed assets is advantageous. A worrisome situation arises when proponents of large ships view the construction of small, expendable ships as a threat to their own survival, and the opposite applies as well. Both systems have their own merits and strengths and what is required is a well balanced mix, developed and refined by generous amounts of research and analysis. The reader is left to balance the merits and demerits of the system and left to make his/her own conclusions.

C. ENDNOTE

This thesis ends with a quote from CAPT. Wayne Hughes. We quote from his article [Ref. 10] written twenty years ago, in 1981.

> *"Let us devise a war game that is simple, replicative, and with characteristics that are understood by most military men. Let us play chess. ...... For the next several years at least, the U.S Navy will have its offensive strength heavily concentrated in its carriers. It is a queenly strength unparalleled in range, sustained destructive power, and mobility. But we have 13 queens and 300 other pieces...(which are) ...substantially less well armed for offense than ... the bishops, knights, and pawns.......One does not have to play missile chess[2] to sense that the White Chessmen so armed will lose. Their offensive power has been over-concentrated. "*

— CAPT. Wayne Hughes

Twenty years have passed since this article. Crossbow aims to provide the White Chessmen with its badly needed bishops, knights and pawns.

[2]Missile Chess is played with normal chess rules and normal chess pieces, with exception that each game piece can only make a limited number of captures. For example, a Queen might be limited to only 10 captures, after which she is unable to capture any more pieces. When all power is transferred to the Queen, the opponent will invariably focus all efforts to capture the Queen. Once the Queen is lost, the game is as good as over as the other pieces have transferred all their power to the Queen.

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APPENDIX A. GLOSSARY

ASCM : Acronym, Anti-Ship Cruise Missile.

Center of Gravity : The part of the fighting force which if destroyed would significantly reduce the combat potential.

Combat Consumable : A description of part of the fleet whose loss can be sustained by the entire fleet, and not jeopardize the overall mission.

Combat Potential : Combat potential is a measure of latent ability. Example: A magazine of missiles on board a ship represents the combat potential of the ship. When the ship launches the missiles towards a target, it converts the latent combat potential into realizable combat power. Similar to the concept of electrical potential energy being stored in battery. The use of the battery would represent transformation of electrical potential energy into electrical power.

Concentrated/Saturation Attack : A form of attack where the enemy launches a large number of missiles at a particular ship of the fleet. This term is usually used to describe the situation where the enemy is able to focus all his available offensive firepower onto a single or a few ships of the fleet to ensure that the ship or ships is/are put out of action.

Crossbar Switching System : A connection configuration where every input is connected to every output in a crossbar matrix fashion.

Distribution : Refers to the sharing of a fixed amount of firepower among a fleet.

Distributed Fleet/Force : *Adjective*, the degree to which the firepower is shared amongst a fleet.*Antonym*: Concentrated Fleet/Force. A distributed fleet has a small percentage of total fleet firepower carried per ship, whereas a concentrated fleet will have a significantly larger percentage of total fleet firepower carried per ship. Example: A fleet has a total of 500 missiles in total. A concentrated fleet would have only 2 ships carrying the 500 missiles, each carrying 50% of the overall load. A distributed fleet will have 50 ships, each carrying 2% of the overall load.

Force Effectiveness : The measure of performance of a fleet capability to conduct battle. It consists of two sub-measures of performance. The first is the number of enemy vessels that the fleet is able to put out of action, the second is the number of own ships that survive the engagement. A fleet with a high force effectiveness is one where it is able to inflict maximum damage on the enemy while sustaining minimal losses.

Leaker : An incoming missile that was not or could not have been shot down by the defense mechanism on board a ship.

Sea Archer : Notional 10,000-ton ship whose main fucntion is to serve as a platform for launching Unmanned Aerial Vehicles.

Sea Lance : TSSE (Total Ship System Engineering, Naval Postgraduate School) designed small surface combatant.

Sea Quiver : Notional logisitic support ship used to support the notional CROSSBOW task force.

Stability : A measure of the change in the combat potential of fleet when a ship/few ships are put out of action. An unstable fleet is one where the loss or departure of a single/a few ships will drastically reduce the combat potential of a fleet.

Streetfighers : The concept of numerous small, fast, but lethal ships to be used mainly for littoral access. Defining characteristics are: Individual ships are able to inflict significant losses on the enemy fleet; Ability to maintain significant offensive firepower even when the fleet suffers a reasonable amount of attrition.

Total Fleet Firepower : The sum of all the firepower carried by a fleet. Example: 10 ships carrying 50 missiles each will have a total fleet firepower of 500 missiles.

APPENDIX B. SIMPLE STOCHASTIC SALVO ANALYSIS - A NUMERICAL EXAMPLE

Refer to Chapter IV, section D, subsection 2 on page 46.

Assume B has 4 ships. Therefore the worth of each ship is $\frac{50}{4} = 12.5$ military units. Total defensive power is 40 missiles. Therefore the defensive power of each ship, b_3, is $\frac{40}{4} = 10$ missiles. Looking up Table V on page 47, the staying power of each ship is 3 missiles.

Number of ships	Military Worth per ship	Defensive Power per ship	Staying Power per ship
4	12.5	10	3

Now assume the enemy arbitrarily targets B with 60 missiles. Assume that the distribution is [12 8 40 0]. This means that 12 missiles are targeted at the first ship, 8 at the second ship, 40 at the third, and none at the last ship. The damage is calculated as follows:

Item	First Ship	Second Ship	Third Ship	Fourth Ship
Incoming	12	8	40	0
Incoming destroyed	10	8	10	0
Leakers	2	0	30	0
Damage	66%	0%	100%	0%
Remaining	33%	100%	0%	100%
Remaining worth	4.1	12.5	0	12.5

The first ship of B has 10 defensive missiles at his disposal, therefore the first ship has managed to shoot down all but 2 of the incoming threats. The first ship is damaged. Its staying power is 3 missiles, therefore after being hit by 2 missiles, it has suffered 66% damage. Its military worth is now only 0.33 x 12.5 = 4.125[1]. The second ship manages to shoot down all 8 incoming missiles and survives intact. The third ship has been the subject of a concentrated attack, is unable to defend against all the incoming missiles and has lost all its value. The fourth ship has not been targeted.

The sum of military worth left after this encounter is 4.125 + 12.5 + 12.5 = 29.125. However this fleet started with 50 units, therefore the fractional loss is $\frac{50-29.125}{50} = 0.4175$. Each encounter is simulated 100 times and the result of a single encounter corresponds to a single dot on the chart.

The model designs each ship to equally likely to be the target of a missile attack in order to simulate the fact that the enemy does not know the worth of each of our ships. From B's point of view, this model is to simulate littoral operations, where the enemy's shore missiles are hidden among the clutter of the land. When the fleet comes within range, a surprise salvo of missiles is launched.

[1]Note that linear degradation of a ships capability is implicitly assumed.

APPENDIX C. ENLARGED GRAPHS FOR SIMPLE STOCHASTIC SALVO

Refer to Chapter IV, section D, subsection 3 on page 48.

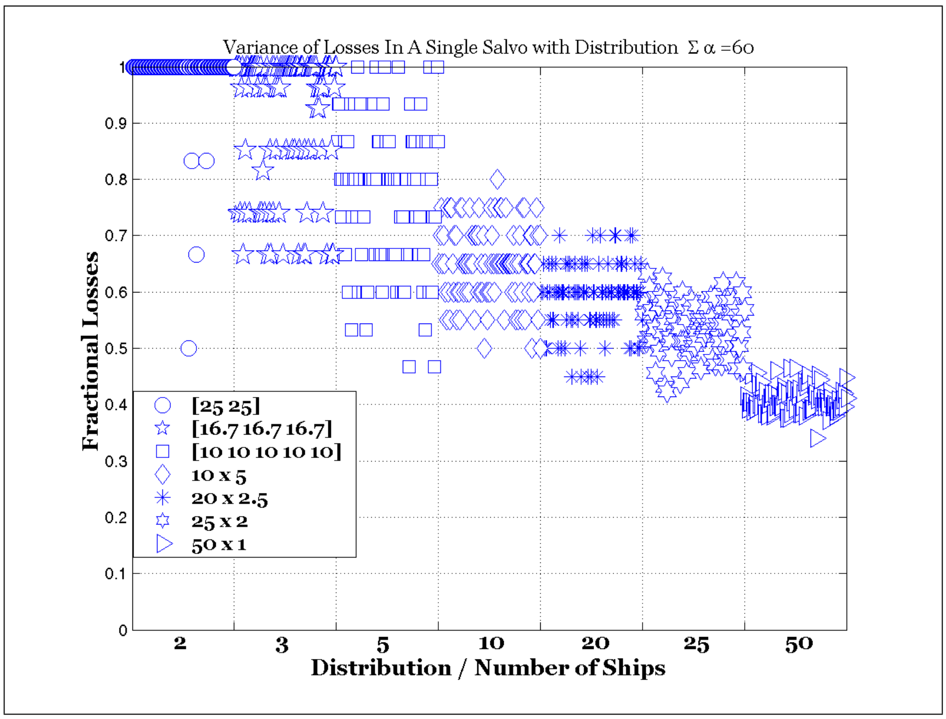

Figure 44. Fractional Loss with 60 Enemy Missiles

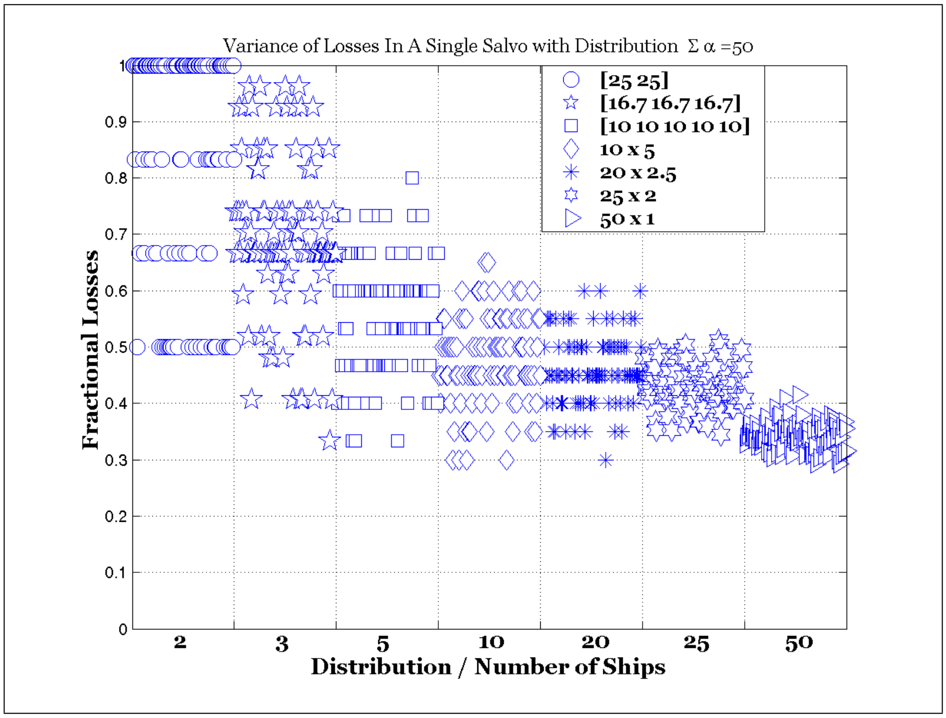

Figure 45. Fractional Loss with 50 Enemy Missiles

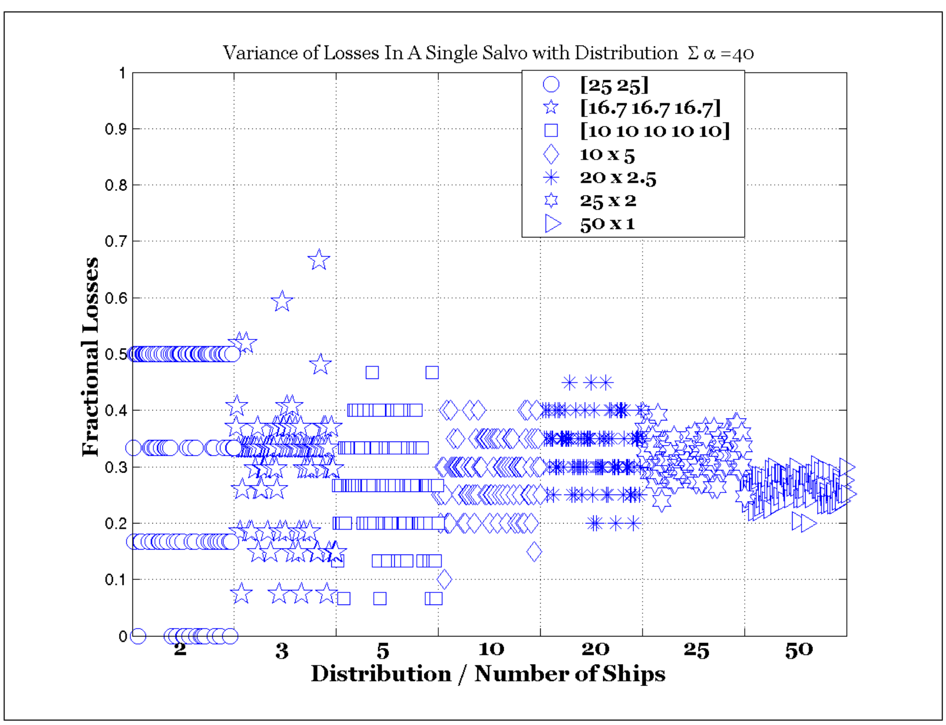

Figure 46. Fractional Loss with 40 Enemy Missiles

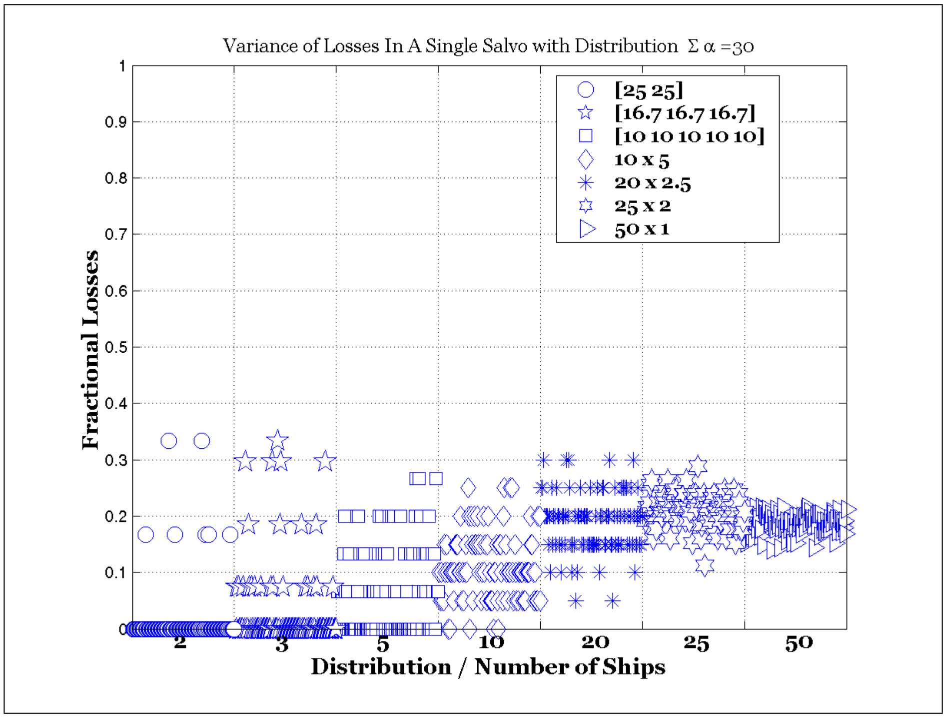

Figure 47. Fractional Loss with 30 Enemy Missiles

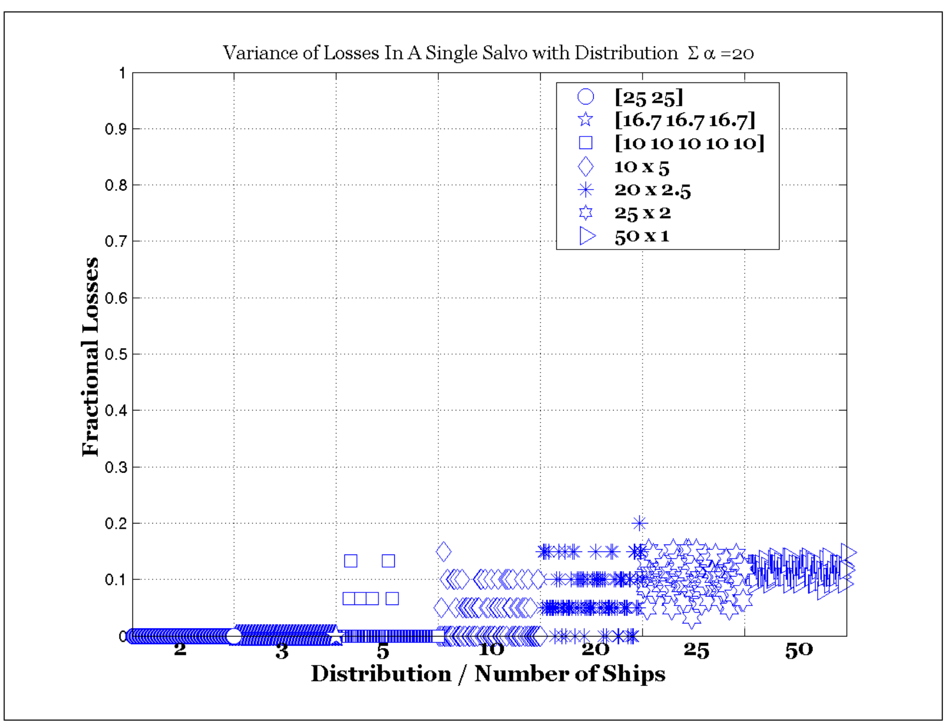

Figure 48. Fractional Loss with 20 Enemy Missiles

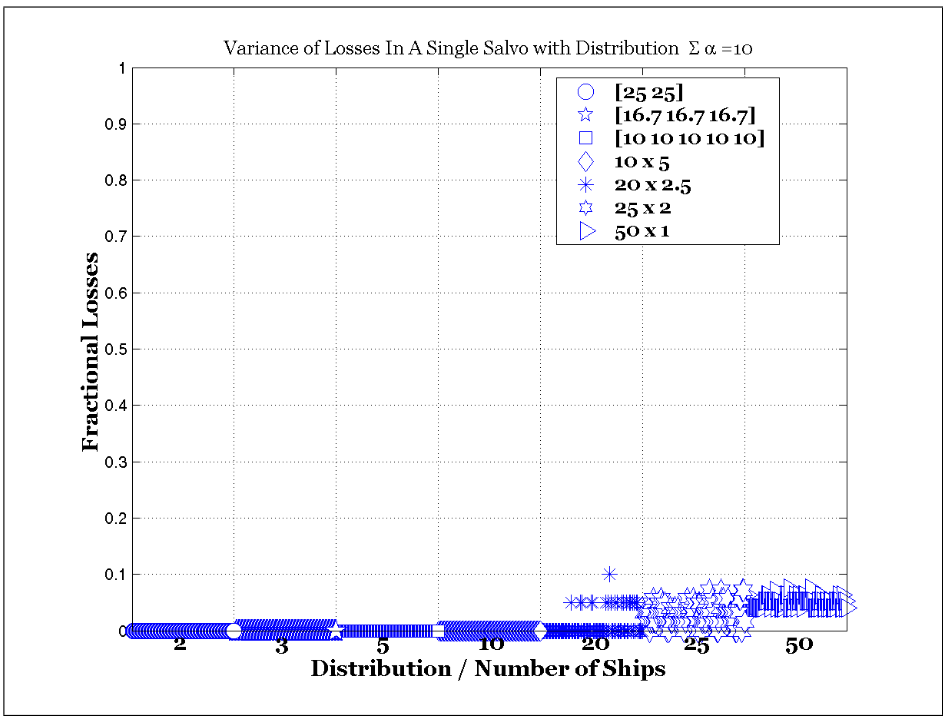

Figure 49. Fractional Loss with 10 Enemy Missiles

APPENDIX D. MATLAB® ALGORITHM FOR STOCHASTIC NAVAL SAVLO MODEL

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This is a stochastic Salvo Equation  %
%Model Based on John McGunnigle's Thesis. %
%The following is the initial information %
%A has I ships, i = 1,2....I %
%B has J ships j=1,2,....J %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear
clc
close all
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%SIMULATION ADMINISTRATION%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%SCOREBOARDKEEPER%%
%%%%%%%%%%%%%%%%%%%%

for J=1:10;
    clear fractionalmonitor1
    clear fractionalmonitor2
figure
totfireB=50;
totdef=40;
%staying power for 1 2 3 4 5 6 7 8 9 10 ships
  stayingpowermat=[3 3 2 2 1 1 1 1 1 1];



Number_of_simulation_runs=10000;
for simcount=1:1:Number_of_simulation_runs;


I=15;   %number of A forces
%J=10;  %number of B forces
aF=4;   %Firepower of A
bF=totfireB/J;   %Firepower of B
aD=1;   %Defensive Readiness ([0,1])
```

```
bD=1;   %Defensive Readiness ([0,1])
aS=1;   %Strike Readiness of A ([0,1])
bS=1;   %Strike Readiness of B ([0,1])
a1=1;   %Staying power of A
b1=stayingpowermat(J);   %Staying power of B
a3=1;   %Defensive Firepower of A
b3=totdef/J;   %Defensive Firepower of B
Aintelfactor=0.8;   %Implies that A knows the
  %status of 50% of B's ships

Bintelfactor=0.8;   %Implies that B knows the
  %status of 50% of A's ships

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Construct the state matrix a for the A forces%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



a=[ones(1,I);    %status of each ship (1st row)
   a3*ones(1,I);  %Defensive Firepower (2nd row)
   aD*ones(1,I);  %Defensive Readiness (3rd row)
   aS*ones(1,I);  %Strike Readiness     (4th row)
   aF*ones(1,I);  %Strike Firepower     (5th row)
   a1*ones(1,I)]; %Staying power        (6th row)
%note that all are assumed to start with 100%

%same for B

b=[ones(1,J);    %status of each ship
   b3*ones(1,J);  %Defensive Firepower
   bD*ones(1,J);  %Defensive Readiness
   bS*ones(1,J);  %Strike Readiness
   bF*ones(1,J);  %Strike Firepower
   b1*ones(1,J)]; %Staying power
%note that all are assumed to start with 100%
%%%%%%%%END OF DATA INPUT%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%WE WILL FIGHT TILL THE LAST SHIP FLOATING!!%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

battle_terminating_switch=0;   %This counter will remain as
 %0 until one side is terminated

salvocounter=0;                      %% First set counter to 0
while battle_terminating_switch<1;  %% The salvo to continue
                 %% until there is a winner
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%HERE COMMENCES THE BATTLE ALGORITHM%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%CALCULATING THE EFFECTIVE FIRE FOR THE NEXT SALVO%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for n1=1:I
ka(n1)=a(1,n1)*a(5,n1); %status of ship * strike firepower
%if k(n1) is not an integer,
%generate a random number from 0 to 1
                        %if the number generated is > decimal
                        %portion round down
                        %if the number generated is < decimal
                        %portion round up

ka2=floor(ka(n1));  %round down to nearest integer
if ka(n1)-ka2>0    %if the number is not an integer
    ka3=ka(n1)-ka2;   % find the decimal portion
    ka4=rand;         %generate the random number
    if ka3>=ka4       %if the decimal portion is
     %greater or equal to the random
     %number
        ka5=ceil(ka(n1)); %round up
        ka(n1)=ka5;
    else
        ka(n1)=ka2;   %if not then round down
```

```
    end
end
end

%repeat the same procedure for fleet B

for n1=1:J
kb(n1)=b(1,n1)*b(5,n1); %status of ship * strike firepower
%if k(n1) is not an integer,
%generate a random number from 0 to 1
                        %if the number generated is > decimal
                        %portion round down
                        %if the number generated is < decimal
                        %portion round up

kb2=floor(kb(n1));  %round down to nearest integer
if kb(n1)-kb2>0    %if the number is not an integer
    kb3=kb(n1)-kb2;     % find the decimal portion
    kb4=rand;          %generate the random number
    if kb3>=kb4         %if the decimal portion is greater or
        %equal to the random number
        kb5=ceil(kb(n1)); %round up
        kb(n1)=kb5;
    else
        kb(n1)=kb2;    %if not then round down
    end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%GENERATING EFFECTIVE NUMBER OF MISSILES FIRED %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


for n1=1:I;                  %to find effective number
     %of missiles launched
    if ka(n1)==0;            %set the platforms with no
 %missiles launched to 0
        ka6(n1)=0;
    else
```

```
       ka6(n1)=ka(n1);      %ka6 will give the number of
         %missiles launched per platform
    end                     %it will be a number of missiles
 %OR 0 if no missiles are launched
end

for n1=1:I;
    if ka6(n1)==0;
        ka7(n1)=0;
    else
        kaholding=0;
        for n2=1:ka6(n1);
            if rand<a(4,n1);
                kaholding=1+kaholding;
            end

         end
         ka7(n1)=kaholding;
    end
end

%ka7 will now give the number of good
%missiles that each platform fires.


for n1=1:J;                 %to find effective
     %number of missiles launched
    if kb(n1)==0;           %set the platforms with no
 %missiles launched to 0
        kb6(n1)=0;
    else
       kb6(n1)=kb(n1);     %ka6 will give the number of
        %missiles launched per platform
    end                    %it will be a number of missiles
%OR 0 if no missiles are launched
end

for n1=1:J;
    if kb6(n1)==0;
        kb7(n1)=0;
    else
        kbholding=0;
```

```
        for n2=1:kb6(n1);
            if rand<b(4,n1);
                kbholding=1+kbholding;
            end
         end
         kb7(n1)=kbholding;
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Random Distribution of Missiles to Enemy Platforms%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Decide if full information is   %%
%given or only partial information is given%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Information given is based on known ships %%
%destroyed. If we assume that  %%
%A has perfect knowledge, he will   %%
%know which ships to target, if not he     %%
%will keep firing at a ship that has   %%
%already been hit and disabled.       %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Therefore we will use the statematrix   %%
%and give an intelligence factor  %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The intelligence factor is a measure of   %%
%how many of the opposite sides  %%
%ship status is known                     %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%Use a new function called intellifun to
%generate the apparent picture as seen by A

B_as_seen_by_A=intellifun(b(1,:),Aintelfactor);
%The row vector gives the information about B
%as how A would see him (1 for still alive, 0 for dead)
%unknowns as classified as still alive

A_as_seen_by_B=intellifun(a(1,:),Bintelfactor);
%The row vector gives the information about A
```

```
%as how B would see him (1 for still alive, 0 for dead)
%unknowns as classified as still alive



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Randomly Allocate the missiles to the enemy%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A_to_B=ranallo(B_as_seen_by_A,sum(ka7));
%The row vector will show A's allocation
%of missiles over B's forces

B_to_A=ranallo(A_as_seen_by_B,sum(kb7));
%The row vector will show B's allocation
%of missiles over A's forces

%%%%%%%%%%%%%%%%%%%%%%
%%%DEFENSIVE FIGHTS%%%
%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%After the offensive launch of missiles,  %
%now B and A have to defend against incoming %
%missiles. We will now calculate how the  %
%defensive battle goes %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%calculating the effective fire for the next salvo%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for n1=1:I
   kda(n1)=a(1,n1)*a(2,n1);

   %status of ship * defensive firepower
   %if k(n1) is not an integer, generate
   %a random number from 0 to 1
   %if the number generated is > decimal
   %portion round down
   %if the number generated is < decimal
   %portion round up
```

```
kda2=floor(kda(n1));      %round down to nearest integer
if kda(n1)-kda2>0         %if the number is not an integer
    kda3=kda(n1)-kda2;    %find the decimal portion
    kda4=rand;            %generate the random number
    if kda3>=kda4         %if the decimal portion is greater
          %or equal to the random number
        kda5=ceil(kda(n1)); %round up
        kda(n1)=kda5;
    else
        kda(n1)=kda2;       %if not then round down
    end
end
end

%repeat the same procedure for fleet B

for n1=1:J
kdb(n1)=b(1,n1)*b(2,n1);

%status of ship * defensive firepower
%if k(n1) is not an integer, generate a
%random number from 0 to 1
   %if the number generated is > decimal
   %portion round down
   %if the number generated is < decimal
   %portion round up

kdb2=floor(kdb(n1));  %round down to nearest integer
if kdb(n1)-kdb2>0    %if the number is not an integer
    kdb3=kdb(n1)-kdb2;  % find the decimal portion
    kdb4=rand;          %generate the random number
    if kdb3>=kdb4       %if the decimal portion is
        %greater or equal to the random number
        kdb5=ceil(kdb(n1)); %round up
        kdb(n1)=kdb5;
    else
        kdb(n1)=kdb2;   %if not then round down
    end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%ACHTUNG!! MISSILE APPROACHING!!%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%CALCULATE HOW MANY OF THE INCOMING WILL BE SHOT DOWN%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%kda gives the number of defensive %%
%missiles fired out by A%   %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%We now calculate how many of these %
%are good shots and the minus off %
%the number of good shots from the  %
%incoming barrage                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for n1=1:I;                      %to find effective number
    %of missiles launched
    if kda(n1)==0;               %set the platforms with no
%defensive missiles launched to 0
        kda6(n1)=0;
    else
        kda6(n1)=kda(n1);       %kda6 will give the number of
  %defensive missiles launched per platform
    end                         %it will be a number of missiles
  %OR 0 if no missiles are launched
end

for n1=1:I;
   if kda6(n1)==0;            %if no defensive missiles are
         %launched
        kda7(n1)=0;             %then the effective number of
              %defensive missiles launched = 0
           %for that platform
    else
       kdaholding=0;           %or else ( first initialize this
         %value known as kdaholding )
        for n2=1:kda6(n1);     %for each defensive missile
 %that that particular platform launches
           randnos=rand;
           if randnos<a(3,n1);     %if the defensive readiness
               %is greater than the random number
                kdaholding=1+kdaholding; %add 1 to the number of good
%defensive missiles launched
            end
```

```
        end                     %the process is repeated for
         %each missile from that platform.
           kda7(n1)=kdaholding; %therefore the number of good
%defensive missile is added now allocated
%to that platform

end                     %the most recent kdaholding will give
%the most updated number of good defensive missiles
end                     %launched. Cycle this process
%for all the ships in the A force

%%%%Do the same thing for B%%%%%%%%%%%%


for n1=1:J;                      %to find effective number of
      %missiles launched
    if kdb(n1)==0;               %set the platforms with no
  %defensive missiles launched to 0
        kdb6(n1)=0;
    else
       kdb6(n1)=kdb(n1);       %kdb6 will give the number of
         %defensive missiles launched per platform
    end                        %it will be a number of missiles
 %OR 0 if no missiles are launched
end

for n1=1:J;
    if kdb6(n1)==0;   %if no defensive missiles are launched
        kdb7(n1)=0;   %then the effective number of defensive
  %missiles launched = 0 for that platform
    else
        kdbholding=0; %or else ( first initialize
  %this value known as kdbholding )
        for n2=1:kdb6(n1); %for each defensive missile that that
  %particular platform launches
            randnos=rand;
           if randnos<b(3,n1);%if the defensive readiness
  %is greater than the random number
                kdbholding=1+kdbholding; %add 1 to the number of good
        %defensive missiles launched
            end
        end            %the process is repeated for each missile
```

```
    %from that platform.
           kdb7(n1)=kdbholding;%therefore the number of good defensive
  %missile is added now allocated to that platform
     end             %the most recent kdbholding will give the
  %most updated number of good defensive missiles
end                  %launched. Cycle this process for all the
  %ships in the B force


%now that we have all the parameters, let us do the salvo exchange

%describe a holding vector for the new status levels
%call it a_new and b_new.

for n1=1:I        % for each ship in A's force
   a_new(n1)=a(1,n1)-((B_to_A(n1) - kda7(n1))/a(6,n1));
    % calculate the status of the
  % ship after engagement
   if a_new(n1)<0;      % if it falls to negative
       a_new(n1)=0;     % (enemy overkill) set the new status to 0

    elseif a_new(n1)>a(1,n1); % if it moves to
               %more than previous value,
                  %(defensive overkill), then limit
                                 % it to original value
       a_new(n1)=a(1,n1);
   end
end                            %repeat for each ship

for n1=1:J                     %for each ship in B's force
   b_new(n1)=b(1,n1)-((A_to_B(n1) - kdb7(n1))/b(6,n1));
     %calculate the status of the
                               %ship after engagement
   if b_new(n1)<0;             %if it falls to negative
          %(enemy overkill) set
          %the new status to 0
       b_new(n1)=0;
   elseif b_new(n1)>b(1,n1);      %if it moves to more
    %than its previous value ,
    %(defensive overkill),
    %then limit it to
                                  %its previous value
```

```
      b_new(n1)=b(1,n1);
   end
end                         %repeat for each ship

fractionalA=sum(a_new)/sum(a(1,:));
fractionalB=sum(b_new)/sum(b(1,:));

if fractionalA==0|fractionalB==0;
battle_terminating_switch=1;
else
   battle_terminating_swtich=0;
end
% a_prev=a(1,:);
% b_prev=b(1,:);
    a(1,:)=a_new;
    b(1,:)=b_new;

    salvocounter=salvocounter+1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%WE WANT TO EXTRACT THE FRACTIONAL%%
%%RATIOS AFTER 1ST AND 2ND SALVO   %%
%%TO INVESTIGATE AFTER INITIAL     %%
%%EXCHANGES WHO HAS THE UPPER HAND %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if salvocounter==1;
    fractionalmonitor1(simcount,:)=[1-fractionalA fractionalB];
end

if battle_terminating_switch==1&salvocounter==1;
    fractionalmonitor2(simcount,:)=[1-fractionalA fractionalB];
elseif salvocounter==2;
    fractionalmonitor2(simcount,:)=[1-fractionalA fractionalB];
end




end %THIS END IS FOR THE WHILE LOOP
```

```
%%%%%%%%%%%%%%%%%%%%%
%%UPDATE SCOREBOARD%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%THE SCOREBOARD READS AS FOLLOWS%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% A WINS %% DRAW %% B WINS %% REMAINING A %% REMAINING B %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if fractionalA==0&fractionalB==0; % if it is a draw
   Scoreboard(simcount,:)= [0 1 0 fractionalA fractionalB];

    elseif fractionalA>fractionalB % if A wins and B loses
      Scoreboard(simcount,:)= [1 0 0 fractionalA fractionalB];
    else % if B wins and A loses
      Scoreboard(simcount,:)=[0 0 1 fractionalA fractionalB];
end

if fractionalB>1;
    break
end

end

A_wins=sum(Scoreboard(:,1));
draws=sum(Scoreboard(:,2));
B_wins=sum(Scoreboard(:,3));

Results1=[A_wins/Number_of_simulation_runs ;
    draws/Number_of_simulation_runs ;
   B_wins/Number_of_simulation_runs];
Results=Results1';
%%%%%%%%%%%%%%%%%%%%
%%SETTING THE DATA%%
%%%%%%%%%%%%%%%%%%%%
n5=0;
n6=0;
n7=0;
A_table=[0 0 0 0 0];
B_table=[0 0 0 0 0];
AB_table=[0 0 0 0 0];
```

```
for n4=1:Number_of_simulation_runs;
   if Scoreboard(n4,1)== 1;
      n5=n5+1;
      A_table(n5,:)= Scoreboard(n4,:);
   elseif Scoreboard(n4,2)==1;
      n6=n6+1;
      AB_table(n6,:)= Scoreboard(n4,:);
   elseif Scoreboard(n4,3)==1;
      n7=n7+1;
      B_table(n7,:)= Scoreboard(n4,:);
   end
end


%%%%%%%%%%%%%%%%%%%%%%
%%PLOTTING THE GRAPH%%
%%%%%%%%%%%%%%%%%%%%%%


subplot(2,2,1:2),
bar(1:3,[Results(1)*100 0 0],'r')
hold on
bar(1:3,[0 Results(2)*100 0],'g')
hold on
bar(1:3,[0 0 Results(3)*100],'b')
grid
title(['Winning Percentage in ',int2str(
Number_of_simulation_runs),' simulations'],
'FontName','Georgia','FontSize',12)

xlabel(['1=A Wins , 2=Draw , 3=B Wins , B='num2str(J)
,', \beta=',num2str(bF),', b1=',num2str(b1)],
'FontName','Georgia','FontSize',12)

ylabel('%','FontWeight','bold',
'FontName','Georgia','FontSize',12)

xlim([0.5 3.5])
ylim([0 100])
if A_table(1,:)==[0 0 0 0 0];
subplot(2,2,3),
plot(1,1);
```

```
xlabel('Red ALWAYS LOSES!!','FontWeight',
'bold','FontName','Georgia','FontSize',12)

ylabel('Frequency','FontName','Georgia'
,'FontSize',12)

xlim([0 1])
hold on
grid
else

avgfracA=sum(A_table(:,4))/sum(A_table(:,1));

subplot(2,2,3),
[h1,h2]=hist(A_table(:,4));
bar(h2,h1,'r')
xlabel(['Frac Red Surv (Red Wins) Avg=',
num2str(avgfracA)],'FontName','Georgia','FontSize',9)

ylabel('Frequency','FontName','Georgia','FontSize',12)

xlim([0 1])
hold on
grid
end

if B_table(1,:)==[0 0 0 0 0];
subplot(2,2,4)
plot(1,1)
ylabel('Frequency','FontName',
'Georgia','FontSize',12)

xlabel('Blue ALWAYS LOSES!!','FontWeight',
'bold','FontName','Georgia','FontSize',12)

grid
xlim([0 1])
else
avgfracB=sum(B_table(:,5))/sum(B_table(:,3));

if avgfracB>1;
```

```
    break
end

subplot(2,2,4)
[h3,h4]=hist(B_table(:,5));
bar(h4,h3,'b');
ylabel('Frequency','FontName','Georgia',
'FontSize',12)

xlabel(['Frac Blue Surv (Blue Wins)
Avg=',num2str(avgfracB)],'FontName','Georgia',
'FontSize',9)

grid
xlim([0 1])
end
hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%PLOTTING THE GRAPHS FOR INITIAL%%
%%FIRST AND SECOND SALVOS        %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure
subplot(2,2,1)
plot(fractionalmonitor1(:,1)
,fractionalmonitor1(:,2),'.');

title('Frac B Surv after 1 Salvo','FontWeight',
'bold','FontName','Georgia','FontSize',10)

ylabel('Frac A Killed after 1 Salvo','Fontweight'
,'bold','FontName','Georgia','FontSize',10)

grid
xlim([0 1])
ylim([0 1])
subplot(2,2,2)
plot(fractionalmonitor2(:,1),
fractionalmonitor2(:,2),'.');

title('Frac B Surv after 2 Salvos',
```

```
'FontWeight','bold','FontName','Georgia','FontSize',10)

ylabel('Frac A Killed after 2 Salvos','FontWeight'
,'bold','FontName','Georgia','FontSize',10)

grid
xlim([0 1])
ylim([0 1])

subplot(2,2,3:4),
bar(1:3,[Results(1)*100 0 0],'r')
hold on
bar(1:3,[0 Results(2)*100 0],'g')
hold on
bar(1:3,[0 0 Results(3)*100],'b')
grid
title(['Winning Percentage In ',int2str
(Number_of_simulation_runs),' Simulations'],
'FontName','Georgia','FontSize',12)

xlabel(['1=A Wins , 2=Draw , 3=B Wins ,
B='num2str(J) ,', \beta=',num2str(bF),',
 b1=',num2str(b1)],'FontName','Georgia',
 'FontSize',12)

ylabel('%','FontWeight','bold',
'FontName','Georgia','FontSize',12)

xlim([0.5 3.5])
ylim([0 100])

Number_of_Runs = J
ScoringTable(J,:)=[A_wins draws B_wins J];
end
theoutput='A wins , draws, B Wins, Number of B Platforms';
theoutput
ScoringTable
```

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APPENDIX E. NUMERICAL EXAMPLE OF "INTELLIGENCE FACTOR"

Refer to Chapter IV, section E, subsection 1 on page 55.

The following example is the use of the "intelligence factor" being demonstrated in a single salvo,

- Assume that there are two sides to the conflict, A and B;
- Assume that A has 3 ships and B has 5 ships;

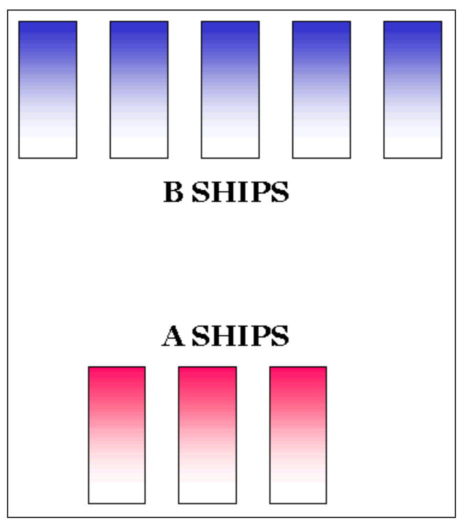

Figure 50. A and B Before Missile Exchange

Figure 50 shows the boats before the missile exchange. Assume that after the missile exchange, B has lost 2 ships and A has lost none, as depicted in Figure 51. Now assume that A only has a 0.6 intelligence factor. This means that out of the 5 ships that B had, A can only know the status of 3 of B's ships. Now the 3 ships that A is able to gain information about is a random selection of 3 ships from the original 5 ships. Each of the 5 original ships has an equal probability of being in this group of 3 ships that A is able to gain information about. Figures 52 and 53 are some possibilities.

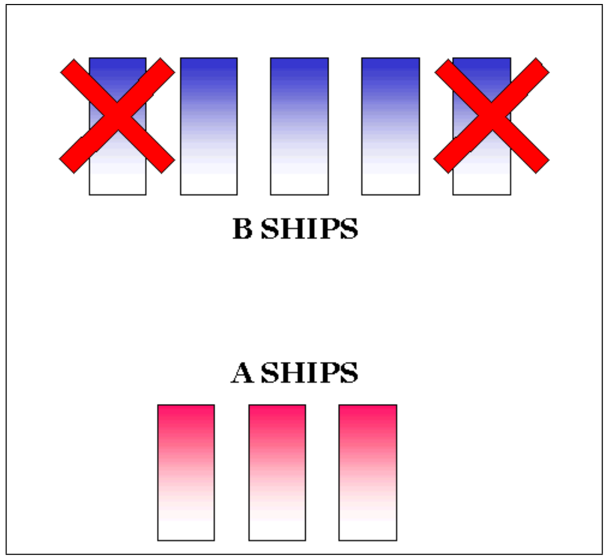

Figure 51. A and B After Missile Exchange

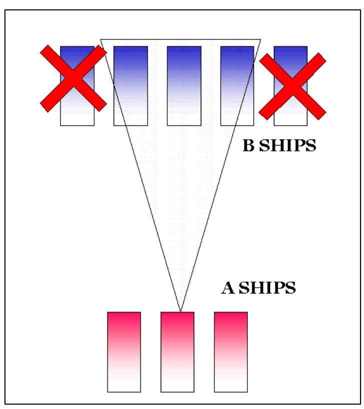

Figure 52. $A_{\text{intelligence factor}}$=0.6

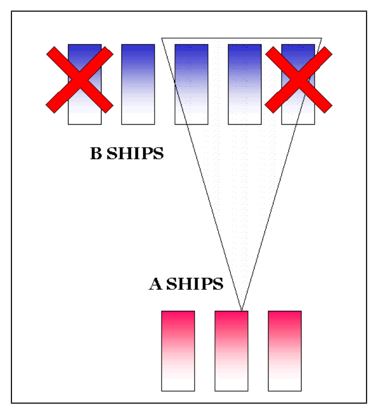

Figure 53. $A_{\text{intelligence factor}}$=0.6

In the first case, Figure 52, A is able to see the 3 B ships that were undamaged and is unable to see the 2 ships that were put out of action. Therefore, in A's next salvo, A will continue to target ALL of B's ships. In the second case, Figure 53, A has information that he has put 1 of B's ships out of action. Therefore in this case, A's next salvo would only target 4 of B's ships (The 2 ships that A knows to be still operational and the 2 that A is unable to gain information about). Note that for each proceeding salvo, the ships that the enemy is able to gain information is about, does not remain the same. That is, if the enemy is able to gain some battle damage assessment about a particular ship after a single salvo, he may not gain battle damage assessement for that same ship in the next salvo.

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